Seminario de la Red de Algebra no Conmutativa.

Almería, 24 y 25 de abril. Departamento de Matemáticas. Universidad de Almería

Red de Investigación del Ministerio de Ciencia e Innovación RED2022-134631-T

Programa

Jueves, 24 de abril. Seminario Emmy Noether (sala 1.22, edf. CITE III)

12-13. Michel Dubois-Violette (Université Paris-Saclay, Orsay Cedex, France)

Complex structures and Spinors.

13-14. Manuel Saorín (Universidad de Murcia)

On a problem raised by Cuadra and Simson

Viernes, 25 de abril. Seminario Emmy Noether (sala 1.22, edf. CITE III)

11-12. Lars Winther Christensen. (Texas Tech University Lubbock, TX.)

Acyclic complexes and regular rings.

12-12:45. Yusuf Alagöz (Hatay Mustafa Kemal University, Hatay, Turkey)

On simple-injective and min-injective modules

12:45-13:45. Sergio Estrada (Universidad de Murcia)

Homotopy equivalences over rings of finite Gorenstein weak global dimension.

ABSTRACTS

Michel Dubois-Violette, *Complex structures and Spinors*

Each isometric complex structure on a 2*l*-dimensional euclidean space E corresponds to an identification of the Clifford algebra of E with the canonical anticommutation relation algebra for *l* (fermionic) degrees of freedom. The simple spinors in the terminology of E. Cartan or the pure spinors in the one of C. Chevalley are the associated vacua. The corresponding states are the Fock states (i.e.pure free states), therefore, none of the above terminologies is very good.

Manuel Saorín, *On a problem raised by Cuadra and Simson*

In a paper of 2007 Juan Cuadra and Daniel Simson asked whether a locally finitely presented Grothendieck category G with enough flats has enough projectives. In an ongoing joint work with Carlos Parra and Lorenzo Martini, we have proved that such a G is AB6 and AB4*, which, by a little variation of a result of Roos, means that G is equivalent to M od − A/T, where A is a small preadditive category and T is a TTF class in M od − A. In particular, by (a generalization of) a result of Jans, there is a uniquely determined idempotent ideal I of A such that T consists of the right A-modules (=additive functor $A^{op} \to$ Ab) annihilated by I. We will show in the talk that Cuadra-Simson problem is then a particular case of a classical problem, first apparently raised when A is a ring (=preadditive category with just one object) by Miller in 1975. This problem asks when an idempotent ideal is the trace of a projective A-module and, as shown by Krause in 2000, also includes the verification-refutation of the telescope conjecture for a given compactly generated triangulated category as a particular case.

We will end the talk by showing some affirmative answers to Cuadra-Simson problem.

Lars Winther Christensen. *Acyclic complexes and regular rings.*

Following Bertin we say that a ring R is regular if every finitely generated ideal I in R has finite projective dimension. In a paper from 2009 Iacob and Iyengar show that a Noetherian ring R is regular if and only if every acyclic complex of injective R-modules is contractible, and they obtain similar

characterizations of regular rings in terms of acyclic complexes of projective and flat modules. More recently, Iacob and Gillespie have obtained similar characterizations of coherent regular rings. In the talk I will discuss how these various conditions on acyclic complexes are equivalent without reference to the regularity property of the underlying ring and add more equivalent conditions for coherent regular rings that accurately resemble the Noetherian setting.

Sergio Estrada. *Homotopy equivalences over rings of finite Gorenstein weak global dimension.*

In 2006, Iyengar and Krause showed that over a commutative noetherian ring with a dualizing complex, there exists a triangle-equivalence between the homotopy category of projectives and the homotopy category of injectives given by tensoring with the dualizing complex. Later, in 2010, Chen showed that for a left Gorenstein ring (that is, a ring of finite Gorenstein global dimension), there exists a triangle-equivalence between the homotopy category of Gorenstein projective modules and the homotopy category of Gorenstein injective modules that restricts to a triangle-equivalence between the homotopy category of projective modules and the homotopy category of injective modules. In the case of commutative Gorenstein rings, Chen's equivalence extends Iyengar and Krause's equivalence up to a natural isomorphism. Chen's proof crucially uses the fact that the Gorenstein global dimension is finite. In this talk, we will generalize Chen's equivalence to rings of finite Gorenstein weak global dimension. This class of rings strictly includes all left Gorenstein rings.

The talk is based on joint work with Junpeng Wang.

On simple-injective and min-injective modules

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Abstract:
Quasi-Frobenius rings were introduced by Nakayama [3] in the study of representations of algebras. Afterwards, Quasi-Frobenius rings played a central role in ring theory, and numerous characterizations were given by various authors. In particular, Ikeda [2] characterized these rings as two sided self-injective and two sided Artinian. Numerous investigations have been conducted to improve Ikedas mentioned result by weakening either the Artinian condition, or the injectivity condition, or both, and this has led to the discovery of new concepts of rings and modules such as simple-injectivity and min-injectivity of rings and modules [1, 4, 5]. Nicholson and Yousif [4], dealt with the question of when a simple-injective ring becomes injective. They show that a semiprimary right simple-injective ring is injective. This result generalizes Osofsky's [6] well-known result that "a left perfect left-right injective ring is a quasi Frobenius ring" to "a left perfect left-right simple-injective ring is a quasi Frobenius ring". Later, Nicholson and Yousif approached the Osofsky's result with min-injective modules that are more general than injectivity and showed in their article [5] that a right Artinian two sided min-injective ring is quasi Frobenius. These two basic approaches are concerned with when simple injective (or min-injective) rings become injective. Inspired by one of this approach, in [1], it is shown that the rings whose each simple-injective right module is injective (resp. projective) are right Artinian (resp. quasi Frobenius) rings. As a natural continuation of aforementioned approaches, in this talk, after summarizing the above results, I will talk about various examples and properties of the rings over which (finitely generated) min-injective right modules are injective and projective, respectively.

This is joint work with S. Benli-Göral, E. Büyükaşık, J. R. Garcia Rozas and L. Oyonarte.

Acknowledgments: This study is supported by The Scientific and Technological Research Council of Turkey (TUBITAK) under the 2219 - International Postdoctoral Research Fellowship Program for Turkish Citizens.

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