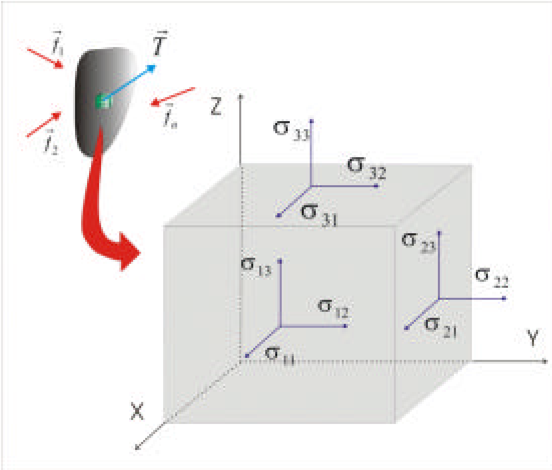
formulario 2: Tensiones

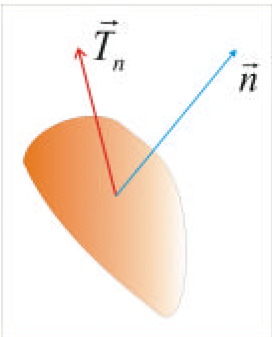
ESTADO TENSIONAL

1. Tensor de Tensiones



$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} = \begin{pmatrix} \sigma_{x} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{y} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{z} \end{pmatrix}$$

2. Ecuación de Cauchy



$$\vec{T} = \sigma^{t}\vec{n} \quad ó \quad T_i = \sigma_{ji} n_j$$

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$

3. Ecuaciones de equilibrio interno

$$\left.\begin{aligned} \frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{21}}{\partial x_2} + \frac{\partial \sigma_{31}}{\partial x_3} + f_1 = 0 \\ \frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{23}}{\partial x_3} + f_2 = 0 \\ \frac{\partial \sigma_{31}}{\partial x_1} + \frac{\partial \sigma_{32}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3} + f_3 = 0 \end{aligned}\right\} \Rightarrow \frac{\partial \sigma_{ij}}{\partial x_i} + f_j = 0 \Rightarrow \sigma_{ij,j} + f_j = 0$$

4. Simetría del tensor de tensiones

$$\sigma_{ij} = \sigma_{ji}$$

TENSIONES Y DIRECCIONES PRINCIPALES

1. Representación del tensor de tensiones en las direcciones principales

$$s_{ij} = \begin{pmatrix} s_I & 0 & 0 \\ 0 & s_{II} & 0 \\ 0 & 0 & s_{III} \end{pmatrix}$$

2. Invariantes I del tensor de tensiones

$$I_1(s) = s_{11} + s_{22} + s_{33} = s_I + s_{II} + s_{III}$$

$$I_2(s) = s_{11}s_{22} + s_{22}s_{33} + s_{33}s_{11} - s_{12}s_{21} - s_{13}s_{31} - s_{23}s_{32} =$$
$$= s_I s_{II} + s_{II} s_{III} + s_{III} s_I$$

$$I_3(s) = \begin{vmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{vmatrix} = s_I s_{II} s_{III}$$

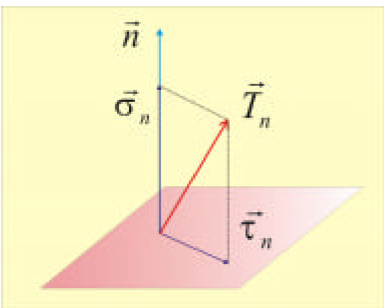
3. Invariantes J del tensor de tensiones

$$J_1(s) = I_1 = s_{ii}$$

$$J_2(s) = \frac{1}{2}\left(I_1^2 + 2I_2\right) = \frac{1}{2} s_{ij} s_{ji}$$

$$J_3(s) = \frac{1}{3}\left(I_1^3 + 3I_1 I_2 + 3I_3\right) = \frac{1}{3} s_{ij} s_{jk} s_{ki}$$

COMPONENTES INTRÍNSECAS DEL VECTOR TENSIÓN



$$\vec{T}_n = \vec{s}_n + \vec{t}_n \quad \Rightarrow \quad T_n^2 = s_n^2 + t_n^2$$

$$s_n = \vec{T}_n \cdot \hat{n}$$

$$t_n^2 = T_n^2 - s_n^2$$

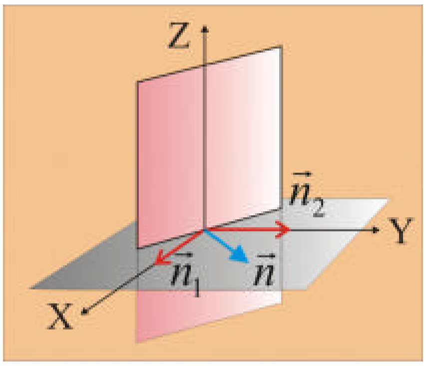
TENSIONES MÁXIMAS

1. Tensión normal máxima

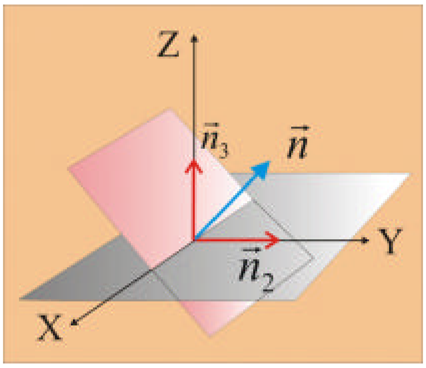
$$s_n^{máx} = T_n$$

2. Tensiones tangenciales máximas

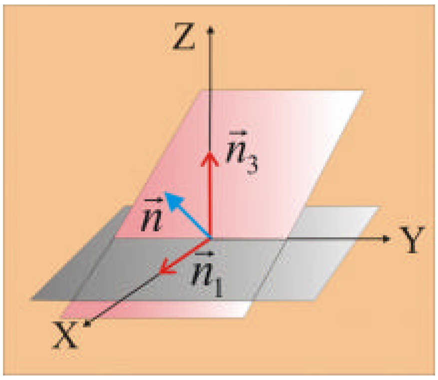
$$t_n^{máx} = \pm \frac{s_I - s_{II}}{2}$$



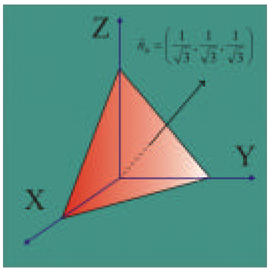
$$t_n^{máx} = \pm \frac{s_{II} - s_{III}}{2}$$



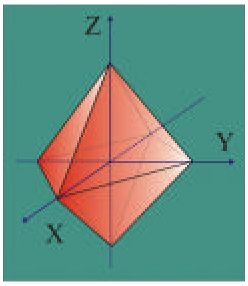
$$t_n^{máx} = \pm \frac{s_I - s_{III}}{2}$$



TENSIONES OCTAÉDRICAS



$$s_{no} = \frac{1}{3}\left(s_I + s_{II} + s_{III}\right)$$



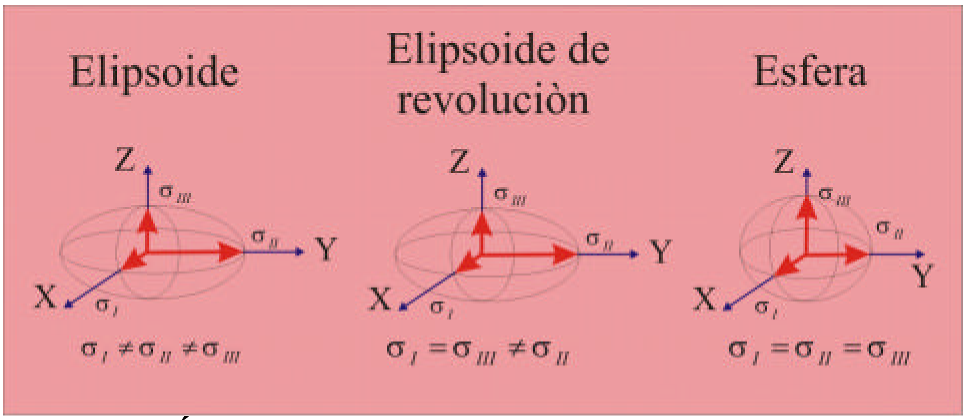
$$t_{no} = \frac{1}{3}\sqrt{\left(s_I - s_{II}\right)^2 + \left(s_{II} - s_{III}\right)^2 + \left(s_I - s_{III}\right)^2}$$

REPRESENTACIÓN OCTAÉDRICO-DESVIADOR

$$s = s_{oct} + s_{des} \quad \begin{cases} s_{oct} = \begin{pmatrix} s_{no} & 0 & 0 \\ 0 & s_{no} & 0 \\ 0 & 0 & s_{no} \end{pmatrix} \\ s_{des} = \begin{pmatrix} s_{11} - s_{no} & s_{12} & s_{13} \\ s_{21} & s_{22} - s_{no} & s_{23} \\ s_{31} & s_{32} & s_{33} - s_{no} \end{pmatrix} \end{cases} \Rightarrow \quad s_{ij} = s_{no} d_{ij} + s_{des\,ij}$$

REPRESENTACIÓN DE LAMÉ

$$\frac{x^2}{s_I^2} + \frac{y^2}{s_{II}^2} + \frac{z^2}{s_{III}^2} = 1$$



REPRESENTACIÓN DE MOHR

1. Círculos de Mohr 3D

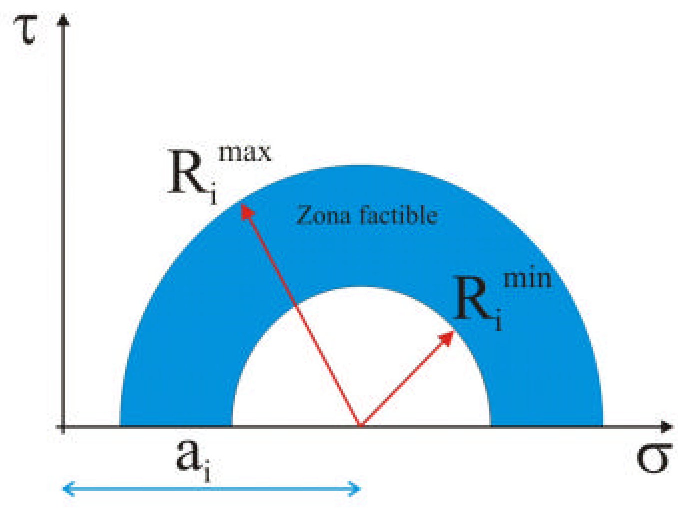
$$A \equiv (s_I - s_{II})(s_{II} - s_{III})(s_I - s_{III})$$

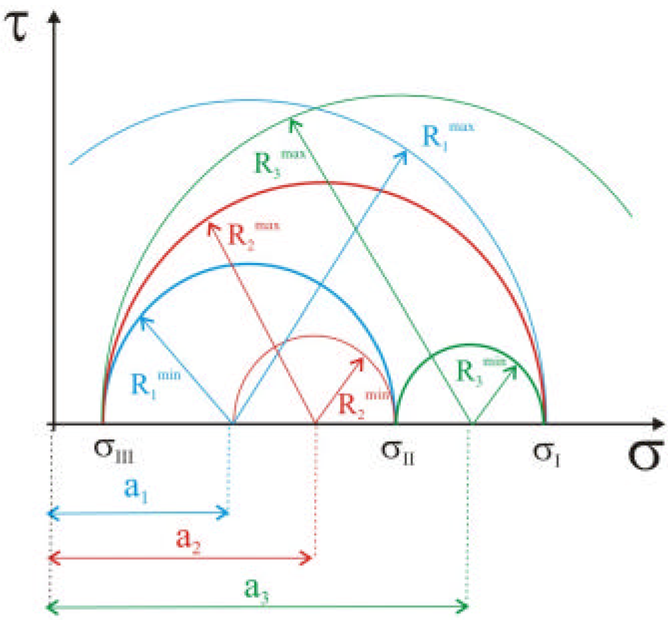
$$\hat{n} = (n_1, n_2, n_3)$$

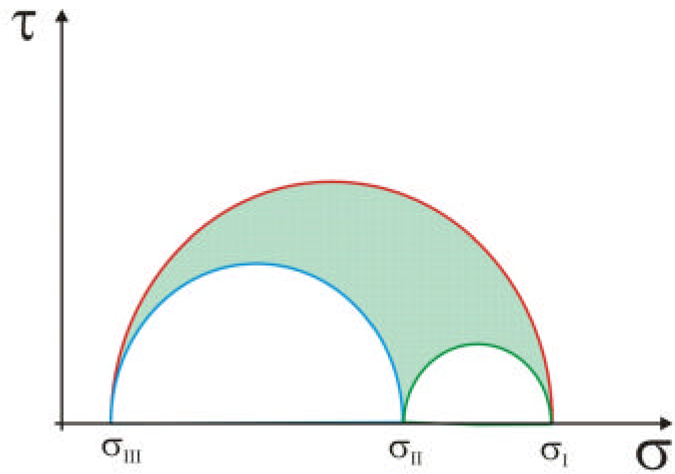
$$t_n^2 + (s_n - a_1)^2 = R_1^2 \quad \begin{cases} a_1 \equiv \dfrac{s_{II} + s_{III}}{2} \\ R_1^2 \equiv \left(\dfrac{s_{II} - s_{III}}{2}\right)^2 + \dfrac{A}{s_{II} - s_{III}} \cdot n_1^2 \quad \begin{cases} R_1^{\min} = \dfrac{1}{2}(s_{II} - s_{III}) \\ R_1^{máx} = \left| s_I - a_1 \right| \end{cases} \end{cases}$$

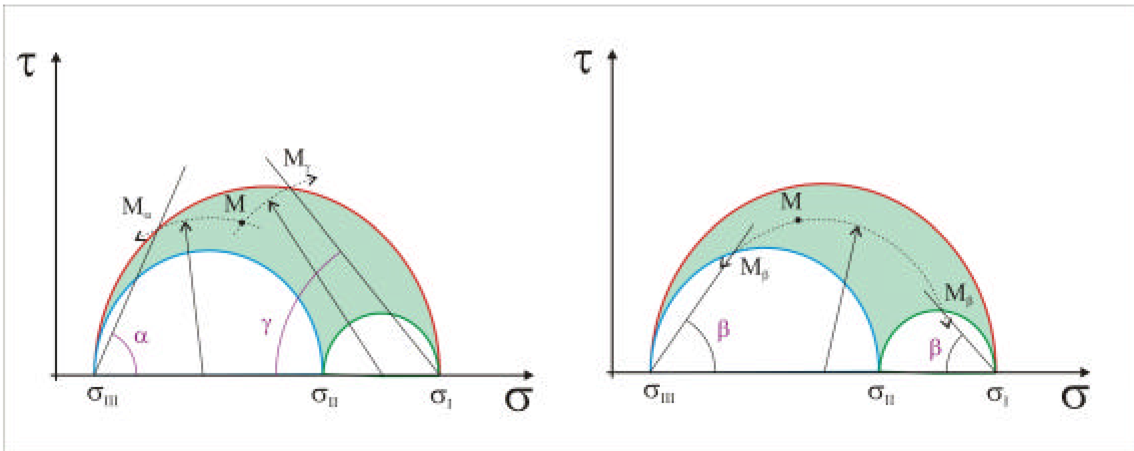
$$t_n^2 + (s_n - a_2)^2 = R_2^2 \quad \begin{cases} a_2 \equiv \dfrac{s_I + s_{III}}{2} \\ R_2^2 \equiv \left(\dfrac{s_I - s_{III}}{2}\right)^2 - \dfrac{A}{s_I - s_{III}} \cdot n_2^2 \quad \begin{cases} R_2^{mín} = \left| s_{II} - a_2 \right| \\ R_2^{máx} = \dfrac{1}{2}(s_I - s_{III}) \end{cases} \end{cases}$$

$$t_n^2 + (s_n - a_3)^2 = R_3^2 \quad \begin{cases} a_3 \equiv \dfrac{s_I + s_{II}}{2} \\ R_3^2 \equiv \left(\dfrac{s_I - s_{II}}{2}\right)^2 + \dfrac{A}{s_I - s_{II}} \cdot n_3^2 \quad \begin{cases} R_3^{\min} = \dfrac{1}{2}(s_I - s_{II}) \\ R_3^{máx} = \left| s_{III} - a_3 \right| \end{cases} \end{cases}$$


τ
R_i^{max}
Zona factible
R_i^{min}
a_i
σ


τ
R_1^{max}
R_3^{max}
R_2^{max}
R_1^{min}
R_2^{min}
R_3^{min}
σ_{III}
σ_{II}
σ_I
σ
a_1
a_2
a_3


τ
σ_{III}
σ_{II}
σ_I
σ

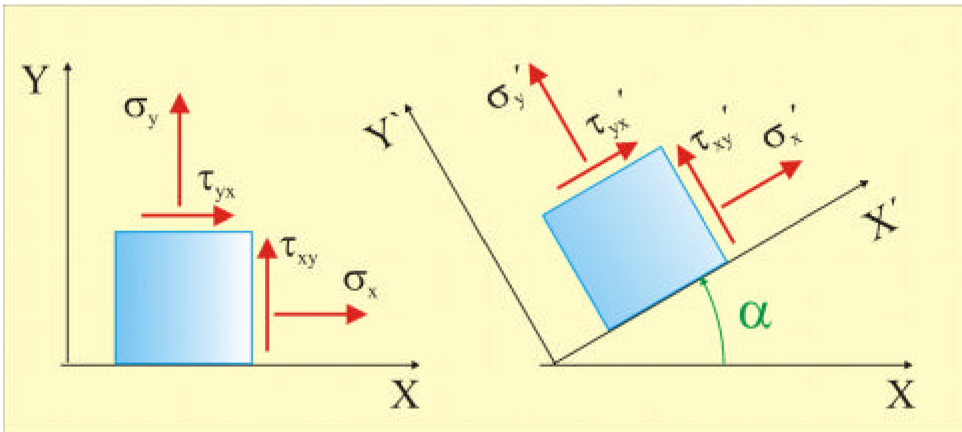


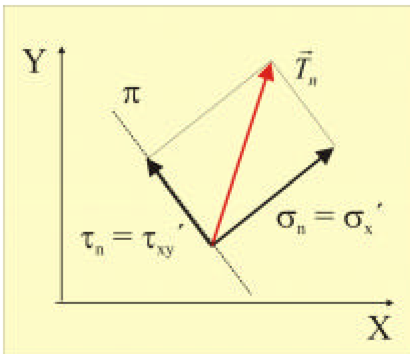
2. Círculos de Mohr 2D

$$s_x^{'} = \frac{s_x + s_y}{2} + \frac{s_x - s_y}{2}\cos(2a) + t_{yx} sen(2a)$$

$$t_{xy}^{'} = \frac{s_y - s_x}{2} sen(2a) + t_{yx}\cos(2a)$$

$$s_y^{'} = \frac{s_x + s_y}{2} - \frac{s_x - s_y}{2}\cos(2a) - t_{yx} sen(2a)$$





$$s_n = \frac{s_x + s_y}{2} + \frac{s_x - s_y}{2}\cos(2a) + t_{yx} sen(2a)$$

$$t_n = \frac{s_y - s_x}{2} sen(2a) + t_{yx}\cos(2a)$$

$$\left(\sigma_n - \frac{\sigma_I + \sigma_{II}}{2}\right)^2 + \tau_n^2 = \left(\frac{\sigma_I - \sigma_{II}}{2}\right)^2$$