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Leavitt path algebras of small graphs

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This work deals with the classification problem of isomorphism classes of Leavitt path algebras associated to graphs of up to three vertices (small graphs in our terminology). We shall use a matrix approach based upon adjacency matrices. When dealing with graphs satisfying condition SING (there is at most one edge between two given vertices), one finds it extremely useful to consider adjacency matrices which, in the particular case of graphs satisfying SING, are binary matrices, that is, matrices with entries in the set $\{0, 1\}$. By knowing the adjacency matrix one can compute the socle of the associated Leavitt path algebra as well as the K_0-groups (including its unit-order). The abundance of properties of $L(E)$ which can be investigated directly in the graph E (or equivalently in its adjacency matrix) together with the fact that matrices can be easily handled with computational techniques, are some of the reasons why matrix methods can be successfully exploited in the study of Leavitt path algebras. One of the drawbacks of the adjacency matrix approach is that different matrices can represent the same graph (up to relabeling of vertices). If the matrix B can be obtained from the matrix A by a series of (simultaneous) permutations of rows and columns, then A and B represent essentially the same graph. Thus we are lead to the problem of classifying orbits of the action of the symmetric group S_n on the set of binary $n \times n$ matrices.

Once this has been done, computational tools are also easily applied to eliminate matrices which after a SHIFT process agree. Thus, from the abundant list of binary matrices, after taking one representative of each orbit (under the action of S_n) and eliminating coincident matrix (up to SHIFT process). We get a restricted list of matrices which we can classify according to the socles or to the K_0-groups of the Leavitt path algebras of their associated graphs.