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Normal form problems in the classification theory of division and absolute valued algebras

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Any eight-dimensional absolute valued algebra having a non-zero central idempotent is isomorphic to an isotope $\mathbb{O}_\sigma = (\mathbb{O}, \circ)$ of $\mathbb{O}$, with multiplication given by $x \circ y = \sigma(x)\sigma(y)$ for an orthogonal identity-fixing linear endomorphism $\sigma \in \mathrm{O}(\mathbb{O})$. Moreover, two such algebras $\mathbb{O}_\sigma$ and $\mathbb{O}_\tau$ are isomorphic if and only if there exists and automorphism $\varphi \in \mathrm{Aut}(\mathbb{O})$ such that $\tau = \varphi^{-1}\sigma\varphi$.

While one might be perfectly satisfied with this characterisation, one could also take the problem one step further, and ask for a unique representative for each isomorphism class of these algebras. Finding such a cross-section for the isomorphism classes amounts to finding a normal form for the set $\mathrm{O}(\mathbb{R}^7)$ under the group action

$$\mathrm{O}(\mathbb{R}^7) \times \mathrm{Aut}(\mathbb{O}) \to \mathrm{O}(\mathbb{R}^7),\ \sigma \cdot \varphi = \varphi^{-1}\sigma\varphi. \tag{1}$$

Similarly, the isomorphism classes of real eight-dimensional flexible quadratic division algebras are parametrised by the orbits of the group action

$$\mathrm{Pds}(\mathbb{R}^7) \times \mathrm{Aut}(\mathbb{O}) \to \mathrm{Pds}(\mathbb{R}^7),\ \delta \cdot \varphi = \varphi^{-1}\delta\varphi \tag{2}$$

(here $\mathrm{Pds}(\mathbb{R}^7)$ denotes the set of positive definite linear endomorphisms of $\mathbb{R}^7$).

In the talk, I shall explain how normal forms for the above group actions can be found, using the concept of a Cayley triple in $\mathbb{O}$ to describe the automorphism group.