On the duality of generalized Hopf and Lie algebras

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Michaelis [1] introduced the notion of a Lie coalgebra and proved [2] the following duality property for any Hopf algebra H:

$$P(H^\circ) \cong Q(H)^*.$$

Here P is the functor that assigns to every Hopf algebra the Lie algebra of primitive elements and Q is the dual functor, introduced by Michaelis, that assigns to every Hopf algebra the Lie coalgebra of indecomposables. H° is the Sweedler dual of H and $(-)^*$ is our notation for the vector space dual, that in particular turns a Lie coalgebra structure into a Lie algebra structure.

Since the work of Michaelis, the notion of a Hopf algebra has known several generalizations and different kinds of dualities on the category of (generalized) Hopf algebras have been introduced, in particular to cover the apparent dualities in the theory of quantum groups. The aim of the work that is presented here, is to lift these dualities for Hopf algebras to the Lie algebra level, in the same spirit as Michaelis' theorem.

Bibliography

[1] W. Michaelis, *Lie Coalgebras*. Adv. Math. **38** (1980), 1–54.

[2] W. Michaelis, *The primitives of the continuous linear dual of a Hopf algebra as the dual Lie algebra of a Lie coalgebra*. In: "Lie algebra and related topics (Madison, WI, 1988)", Contemp. Math. **110**, 125–176. Amer. Math. Soc., Providence, RI, 1990.