SUMMER SCHOOL ON STRING TOPOLOGY AND HOCHSCHILD HOMOLOGY; APPLICATIONS TO MATHEMATICAL PHYSICS

University of Almería, September 16–20, 2003

Summaries of courses

String Topology and beyond, by Alexander A. Voronov. String Topology is a new field of Algebraic and Geometric Topology suggested or motivated by Mathematical Physics, in particular, Quantum Field Theory and String Theory. This field was opened to the mathematics community by a pioneering work of Chas and Sullivan, String Topology, to appear in the Annals of Mathematics [4]. Further development came through a number of papers, including those of Chas [3], Cohen and Jones [5], Cattaneo, Froehlich, and Pedrini [2], Kisisel [12], Kaufmann, Livernet, Penner [11], Sullivan and the lecturer [15]. The field is developing rapidly and is at a lucky stage, when it appears that the more the field develops, the more exciting problems it opens.

String Topology uses ideas of sigma-model (a two-dimensional quantum field theory) to introduce new algebraic structure on the homology of a free loop space. This algebraic structure is similar to the celebrated quantum cohomology and Gromov-Witten invariants, but done in a purely topological setting. In a way, in String Topology one constructs fusion rules and the whole structure of a 2d quantum field theory on a loop space. Loop spaces have long been very important objects of Algebraic Topology and studied very extensively. One of the reasons for the current excitement about String Topology is that it tells new things about these old objects. Further study of String Topology had brought and will undoubtedly bring powerful new results in Algebraic Topology and Mathematical Physics.

String Topology, Graphs and Morse Theory, by Ralph Cohen. In these lectures I will survey several aspects of string topology. Topics will be chosen from the following:

- Multiplicative constructions on Thom spectra of bundles over the loop space. The relation to topological Hochschild (co)homology.
- Is string topology a homotopy invariant?
- Applications of spaces of graphs to parameterize string topology operations. What we know and don't know about the field theoretic properties of string topology.

- Operations on open strings.
- A Morse theoretic viewpoint of string topology. Relation to Gromov-Witten theory.

Hochschild and cyclic homology and its relation with free loop spaces, by Kathryn Hess. Hochschild and cyclic homologies are in some strong sense additive versions of algebraic K-theory [13]. In this way computing these homology theories is fundamental. But it is also strongly related to non-commutative geometry and the deep algebraic structure underlying the Hochschild cochain complex (the so called Deligne's conjecture) is one of the main ingredient in Kontsevich's proof of deformation by quantification of Poisson manifolds.
Also motivated by computations in algebraic K-theory, Bokstedt, Hsiang, Madsen have introduced Topological Hochschild and Topological cyclic homology [1]. These two theories have been a wonderful technical tool to compute some K-groups [9].
Recently borrowing ideas from rational homotopy theory Hess and Rognes have given algebraic models for computing Topological Hocschild and cyclic homology. These models are based on previous work on algebraic models for homotopy pullbacks by Dupont and Hess [6] in particular free loop spaces [7]. This is also related to the closed geodesics problem [14].

References

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