A GEOMETRIC INEQUALITY AND ITS APPLICATION

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Let H be a Hilbert space. Suppose that $|||\cdot|||$ is a complete uniform norm on $B(H)$. Then the inequality

$$|||\log(a^{\frac{1}{2}}ba^{\frac{1}{2}})||| \leq |||\log a||| + |||\log b||| \tag{1}$$

holds for every pair $a, b \in B(H)^{-1}_{+}$. Applying this inequality we prove that certain subsets of the positive cone of a unital C^*-algebra is a *generalized gyrovector space* with respect to these norms. We exhibit the form of surjective isometries on these subsets of the poisitve cones with respect to the metric induced by these complete uniform norms.