Rational Bet Approximation on $(-\infty, 0]$

by Herbert Stahl

We consider rational best approximants $r_n^* = r_n^*(f, (-\infty, 0]; \cdot) \in R_{n,n+k}$, on $(-\infty, 0]$, $k \in \mathbf{Z}$ fixed, to functions of the form

$$f = u_0 + u_1 \exp \qquad (*)$$

with u_0, u_1 given rational functions. As usual in rational approximation with free poles, the study of the approximants r_n^* is inseparably tied up with an investigation of orthogonal polynomials, which in this specific case are orthogonal polynomials with varying weights and a non-Hermitian orthogonality relation that lives on a curve in the complex plane.

Starting point of the talk will be the famous solution of the $'1/9'$ problem by Gonchar & Rakhmanov from 1986, which deals with the uniform rational approximation of the exponential function on $(-\infty, 0]$. This result will be extended to functions of type $(*)$, and further some related questions will be addressed, as for instance, overconvergence throughout the complex plane $\mathbb{C}$, the asymptotic distribution of the poles of the approximants, and the convergence of close-to-best approximants. The renewed and extended interest in the by now classical $'1/9'$ problem is motivated by applications in numerical analysis, where good rational approximants are needed for functions of type $(*)$, as for instance, for 'φ functions' that appear in exponential integrators.