S-curves and (non-hermitian) orthogonal polynomials

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Consider a sequence of polynomials (P_n) satisfying the (non-hermitian) complex orthogonality

$$\int_\Gamma z^j P_n(z) e^{-nV(z)} dz = 0, \ j = 0, \dots, n-1,$$

where V is a fixed polynomial and the integration is on an unbounded simple contour Γ in $\mathbb{C}$ ending up at ∞ in both directions and such that $\operatorname{Re} V(z) \to +\infty$, as $z \to \infty$ in Γ.

If the polynomial V is real and $\Gamma = \mathbb{R}$, the zeroes of the P_n's are also real and their limiting distribution can be characterized in terms of an equilibrium problem with external field on the real line. In contrast, if V is no longer real we have a lot of freedom in choosing the contour Γ, and this freedom is reflected in the behavior of the zeroes of the polynomials P_n's.

Gonchar and Rakhmanov [1] characterized the limiting distribution of these zeroes, conditioned to the existence of a curve Γ with a certain symmetry property - the so called *S-property* - over which we can compute the integrals above.

Based on recent works [2,3], we will discuss the existence of this curve Γ and its characterization.

This is a joint work with Arno Kuijlaars.

References

[1] A. A. Gonchar and E. A. Rakhmanov, *Equilibrium distributions and the rate of rational approximation of analytic functions*, Mat. Sb. (N.S.) 134(176) (1987), no. 3, 306–352, 447.

[2] A. Martínez-Finkelshtein and E. A. Rakhmanov, *Critical measures, quadratic differentials, and weak limits of zeros of Stieltjes polynomials*, Comm. Math. Phys. 302 (2011), no. 1, 53–111.

[3] E. A. Rakhmanov, *Orthogonal polynomials and S-curves*, Contemp. Math., vol. 578, Amer. Math. Soc., Providence, RI, 2012.