Determination of certain Quadrature Rules on the Unit Circle and the Frequency Analysis Problem[1]

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Abstract

Let ν be a positive measure on the unit circle, $\mathcal{C}$, such that $d\nu(1/z) = -d\nu(z)$. Thus, the moments $\mu_n = \int_{\mathcal{C}} z^n d\nu(z)$, $n = 0, 1, 2, \ldots$, are all real, the associated monic Szegő polynomials, S_n, $n \geq 0$, are real and that the associated reflection coefficients, $\delta_n = S_n(0)$, satisfy $-1 < \delta_n < 1$, for $n \geq 1$.

We consider the reciprocal polynomials $S_n^*(z) = z^n \overline{S}_n(1/z)$ and the n-point the quadrature rule

$$\int_{\mathcal{C}} f(z) d\nu(z) = \sum_{k=1}^{n} \lambda_{n,k} f(z_{n,k}),$$

based on the zeros $z_{n,k}$, $k = 1, 2, \ldots, n$ of the para-orthogonal polynomials $S_n(z) + S_n^*(z)$, which holds for $f(z) \in \text{Span}\left\{z^{-n+1}, z^{-n+2}, \ldots, z^{n-2}, z^{n-1}\right\}$.

We study some techniques for determining the nodes $z_{n,k}$ and weights $\lambda_{n,k}$ of this quadrature rule. As an application, we consider the frequency analysis problem.

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