ORTHOGONAL POLYNOMIALS AND PARTIAL DIFFERENTIAL EQUATIONS ON THE UNIT BALL

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ABSTRACT. Orthogonal polynomials of degree n with respect to the weight function $W_\mu(x) = (1 - \|x\|^2)^\mu$ on the unit ball in $\mathbb{R}^d$ are known to satisfy the partial differential equation

$$\left[\Delta - \langle x, \nabla \rangle^2 - (2\mu + d)\langle x, \nabla \rangle\right] P = -n(n + 2\mu + d)P$$

for $\mu > -1$. The singular case of $\mu = -1, -2, \ldots$ is studied in this paper. Explicit polynomial solutions are constructed and the equation for $\nu = -2, -3, \ldots$ is shown to have complete polynomial solutions if the dimension d is odd. The orthogonality of the solution is also discussed.