Some equilibrium problems with applications in Approximation theory

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The equilibrium problems in the presence of external fields have a number of applications in several problems in approximation theory; in particular, in the asymptotic study of orthogonal and Heine-Stieltjes polynomials .

Namely, given a closed set $\Sigma \subset \mathbb{C}$ and an admissible external field (in the "Saff-Totik sense" [1]) φ on Σ , the equilibrium measure of Σ in the presence of φ is the unique unit measure μ_{φ} with support supp $\mu_{\varphi} \subset \Sigma$ for which the infimum of

$$I_{\varphi}(\sigma) = -\int \int \log |x - z| d\sigma(x) \, d\sigma(z) \, + \, 2 \int \varphi(x) \, d\sigma(x)$$

is attained. Moreover, $\operatorname{supp} \mu_{\varphi}$ is a compact subset of Σ . But in general it is a difficult task to determine $\operatorname{supp} \mu_{\varphi}$.

In this talk we discuss some simple cases of particular importance in the applications.

References

[1] E.B. SAFF AND V. TOTIK, Logarithmic Potentials with External Fields, volume 316 of Grundlehren der Mathematischen Wissenschaften (Springer-Verlag, Berlin, 1997).