Numerical Generation of the Nodes and Weights of a Gaussian Type Quadrature Rule

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Summary: *A positive measure ψ defined on $[a,b]$ such that its moments $\mu_n = \int_a^b t^n d\psi(t)$ exist for $n = 0, \pm 1, \pm 2, \ldots$, can be called a strong positive measure on $[a,b]$. If $0 \le a < b \le \infty$ then the sequence of (monic) polynomials $\{Q_n\}$, defined by $\int_a^b t^{-n+s} Q_n(t) d\psi(t) = 0$, $s = 0, 1, \ldots, n-1$, is known to exists. We refer to these polynomials as the L-orthogonal polynomials with respect to the strong positive measure ψ. The purpose of this exposition is to consider an interpolatory quadrature rule on the nodes of the polynomials $G_{n+1}(z;w) = Q_n(w)Q_{n+1}(z) - Q_{n+1}(w)Q_n(z)$ and provide the numerical tecniques for the generation of the nodes and weights of these quadrature rules, based on a eigenvalue problem.*

Key-words: *Gaussian Type Quadrature, L-Orthogonal Polynomials, Eigenvalue Problems*

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