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Short communication

Efficient methods to convert LiDAR-derived ellipsoid heights to orthometric heights

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ABSTRACT

Nowadays cartographic products are usually obtained from data sources which provide large amount of data. LiDAR acquisition system is a good example of the great quantity of data obtained, such as points with spatial coordinates in a determined reference system. The height of these points is usually related to a global ellipsoid (e.g. WGS84), but the local vertical reference system, and so the corresponding orthometric heights, are usually measured from a local geoid which is adjusted for a country or region. Orthometric height determination can be performed for each point by knowing the undulation value which relates the ellipsoid to the geoid for each position. However, this operation may not be necessary for all points if we take into account the LiDAR specifications. Thus we can use a simplification which minimizes the processing time for this calculation. In this paper we present the results obtained by applying several simplifications to drastically shorten the number of point-to-point computations to obtain the orthometric height from the raw LiDAR point cloud data.

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1. Introduction

The development of LiDAR (light detection and ranging) technology supposes an alternative to the traditional generation of digital elevation models (DEM) based on applying stereo-matching techniques to photogrammetric images. The LiDAR system allows the direct acquisition of terrain points through a laser scanner which determines the travel time of the laser pulse (time-of-flight measurement). In fact, the measurement of the time delay created by light travelling from a source to a reflective target surface and back to a light detector offers a method of evaluating distance (Beraldin et al., 2010). As a second component, an integrated GPS-IMU module enables to measure exactly the position and orientation of the LiDAR system. In this sense, the acceleration data recorded by the Inertial Measurement Unit (IMU) can be used to support the interpolation of the platform position on the GPS trajectory, while rotation rates recorded by the IMU are used to compute platform orientation. The combination of GPS and IMU data allows one to reconstruct the flight trajectory to an accuracy of better than 10 cm (Beraldin et al., 2010). These data are computed jointly to the distance obtained from the laser scanner, determining the 3D coordinates of each point. This technology is able to generate a very dense and accurate set of points with coordinates related to a GPS

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reference system (WGS84). The reader is referred to Vosselman and Maas (2010) for a more in-depth discussion of topographic LiDAR remote sensing technology.

On the other hand, some national and international European institutions have recently adopted the ETRS89 (European Terrestrial Reference System 89) system as the official reference framework. In Spain, ETRS89 has been the official reference system since 2007 (BOE, 2007). In this way, it is worth noting that the ETRS89 system (based on the GRS80 ellipsoid) is similar to the WGS84. Actually, the WGS84 originally used the GRS80 reference ellipsoid, but has undergone some minor refinements in later editions since its initial publication. Most of these refinements are important for high-precision orbital calculations for satellites but have little practical effect on typical topographical uses (e.g. Hooijberg, 1997). Thus, they can be considered practically the same taking in account the accuracy of LiDAR systems. In this sense, the planimetric coordinates obtained from LiDAR could be deemed as definitive for any project. However, the orthometric heights are not related to the ellipsoid WGS84 but rather to a local geoid determined for a region or country. The geoid is that equipotential surface which would approximately coincide with the mean sea level (MSL) (e.g. Torge, 2001). Where a mass deficiency exists, the geoid will dip below the mean ellipsoid. Conversely, where a mass surplus exists, the geoid will rise above the mean ellipsoid. The deviation between the geoid and an ellipsoid is called the "geoid separation" or "geoid undulation". Each region or country establishes a mean sea level point as height origin or vertical datum,

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which is coincident with the geoid. This is achieved by registering the ocean's water level at coastal places over several years using tide gauges. For example, in Spain the height origin of the National Vertical Reference System is the MSL measured at Alicante tide gauge station (BOE, 2007).

Traditionally, orthometric height determination is performed by levelling techniques using points with known heights (altimetric network). Nowadays, there are some local geoid models presenting high vertical accuracy which allow the use of the derived values of undulations to obtain the corresponding orthometric heights from GPS measured ellipsoid heights. For example, the Spanish vertical reference system is materialized by the Spanish High Precision Levelling Network ('Red de Nivelación de Alta Precisión', REDNAP). The recently published Earth Gravitational Model (EGM2008) geoid model, built by a 5' \times 5' grid of undulation values with a global accuracy of about 15 cm (Pavlis et al., 2008), has been already adapted to the Spanish REDNAP by means of a correction surface adjusted by applying the minimum curvature algorithm over around 13,700 check points where both the orthometric and ellipsoid heights were known. The vertical residuals coming from this adjustment were lower than 7 cm in more than 99% of the cases (IGN, 2009). This EGM08-REDNAP local geoid model is currently distributed in a $1' \times 1'$ grid.

The transformation used for obtaining the orthometric height (H) from the ellipsoid height (h) is given by the following expression:

$$H = h - N \tag{1}$$

where *N* is the geoid undulation. The undulation value is computed by interpolating over the corresponding geoid model grid. This process can be carried out by using several methods such as nearest neighbour, inverse distance squared, bilinear interpolation, etc. (Smith et al., 2003). Bilinear interpolation method is the most frequently used, since it is very easy-to-apply in the case of grid formats (typical of geoid models), showing less computational complexity than other methods (e.g. cubic convolution, inverse distance, etc.) and offering a reasonably accurate results. In this way, there are tools specifically designed to cope with the aforementioned task like that developed by the National Geographic Institute of Spain (IGN, 2010). This tool has not been thought for transforming a great quantity of points because it would require a large processing time. In this work could be fully justified.

However, the easy obtaining of point-to-point transformation from ellipsoid to orthometric heights can become a problem when we are working with several million points, what is very usual in LiDAR projects. In this context, the obtaining of millions of undulation values could be very time consuming and, furthermore, unnecessary if we take into account the vertical accuracy of LiDAR data and the technical specifications demanded in these projects. For example, the LiDAR system Leica ALS60 presents nominal height accuracy close to 15 cm (Leica Geosystems, 2008), while, on the other hand, the specifications of the National Aerial Orthophotographies for LiDAR Flights in Spain (Ministerio de Fomento, 2010) recommend a vertical accuracy of 20 cm regarding the ancillary DEM. In this sense, there is a great quantity of studies which analyze the vertical accuracy of a LiDAR-derived DEM studying different land covers (Hodgson and Bresnahan, 2004; Aguilar and Mills, 2008; Aguilar et al., 2010), or the accordance of a DEM to the ASPRS quality standard (Flood, 2004).

In this paper we present the results of an analysis of different alternatives for making less complex and more efficient the process to obtain orthometric heights from LiDAR data. The main goal of this short communication is to report to the technical and scientific Earth observation community about there is no need, under certain conditions, to be afraid about transforming ellipsoid heights to orthometric ones. Thus, our alternative approaches focus on reducing the number of required points with known undulation while maintaining the corresponding transformation vertical error clearly below the nominal vertical uncertainty offered by LiDAR technology.

2. Methodology and application

The proposed methodology can be applied to any LiDAR project and different data models (point clouds, raster DEMs, etc.) where the final product should contain orthometric heights (usually DEMs). In order to make more efficient the process to convert ellipsoid to orthometric heights, several approaches are investigated such as: (i) the use of only one point with known undulation situated in the centre of the flight strip and assigning this value for the rest of the LiDAR-acquired points ("One known point approach" in Fig. 1a), (ii) the use of a value of undulation obtained by interpolating each point (bilinear interpolation) from the four values obtained from the vertexes of the minimum bounding rectangle (MBR) of the flight strip ("Bilinear interpolation from four boundary points approach" in Fig. 1b), and (iii) the extrapolation of the undulation values for the LiDAR-acquired points from the undulation values calculated for the points situated along the LiDAR trajectory (LiDAR trajectory approach in Fig. 1c). These LiDAR trajectory values are computed by using the point position in the nearest trajectory or using the GPS time to interpolate the undulation as a function of this time.

The results obtained from the three aforementioned approaches are compared with the undulation values computed by using all the available undulation values within the flight strip (i.e., those corresponding to the $1' \times 1'$ grid local geoid model EGM08-REDNAP) and adopting a typical four nearest neighbour bilinear interpolation scheme ("No simplification approach" in Fig. 1d).

The application of each approach obviously requires different computational burden. The approach based on "One known point" is the simplest because it only needs one value of undulation to be obtained at the centre point of the flight strip. The approach called "Bilinear interpolation from four boundary points" is slightly more complex because it requires four values from four points located at the corners of the corresponding rectangular strip and processing of the bilinear interpolation for all LiDAR-acquired points. The third approach, named "LiDAR trajectory", supposes another more difficult step because the undulation is determined with respect to a higher number of points. More specifically, the undulation is calculated for all the points situated along the LiDAR trajectory with GPS position. Subsequently the undulation values for the remaining points are extrapolated. In this case, and using the GPS time, the undulation for each point acquired at any time (t_i) can be calculated by interpolating the undulations $(N_0 \text{ and } N_1)$ given by the previous and the next GPS time along the trajectory (t_0 and t_1 , respectively, in Eq. (2)).

$$N_i = N_0 + (N_1 - N_0) \cdot \frac{t_i - t_0}{t_1 - t_0}$$
⁽²⁾

The control case, i.e., without applying any simplification and, thus, computing undulation for each LiDAR-acquired point by bilinear interpolation from the corresponding four nearest neighbours within the $1' \times 1'$ EGM08-REDNAP grid geoid model, actually supposes the determination of millions of values of undulation. In this study we have used this last approach to evaluate the accuracy performance of the proposed three other approaches or simplifications. In this sense, it could deem to be the ground truth.

This methodology has been applied to two LiDAR projects located at two different areas within Almería province (Southeast Spain) in order to analyze the influence of several factors in J.L. Pérez et al. / International Journal of Applied Earth Observation and Geoinformation 18 (2012) 573-578



Fig. 1. Description of the different tested approaches to compute geoid undulations.

the results. The first project was situated in a flat coastal area (Coastal LiDAR) just over the boundary of the used geoid model. The second one was located at Gador mountain range, a mountainous area inside the Iberian Peninsula (Gador LiDAR) where the geoid undulation shows a higher variability due to the presence of steep mountain slopes. In Fig. 2 is depicted the situation of both projects in Almería Province, as well as a multi-level contour representation of the undulations corresponding to the EGM08-REDNAP geoid model. Fig. 2 (on the right) also shows that the Coastal LiDAR area presents a lower geoid undulation variation within the flight strip than the Gador LiDAR area. The points clouds acquired in both projects took a value of 13,458,497 points and 6,979,848 points for Coastal LiDAR and Gador LiDAR respectively. On the other hand, the Coastal LiDAR comprises an area of about

790 ha (11,250 m \times 700 m) and the Gador LiDAR an area of about 1960 ha (10,900 m \times 1800 m).

The undulation value for each needed point, depending on the tested approach, was obtained by using the EGM08-REDNAP geoid model (IGN, 2009) and applying bilinear interpolation from the four nearest neighbours. All the computation tasks were carried out by a computer tool developed for this work and programmed in C++ environment.

3. Results and discussion

The results obtained are shown in Table 1, which presents statistical values based on the point-to-point differences between the

Table 1

Summary of statistical results corresponding to the distribution of vertical residuals obtained as the differences between each simplified approach and the ground truth. A1: "One known point approach"; A2: "Bilinear interpolation from four boundary points approach"; A3: "LiDAR trajectory approach".

Coastal	Mean [m]	-0.012	0.005	-0.001	NA
LiDAR	Maximum signed vertical residual [m]	-0.128	0.011	0.021	NA
	Standard deviation [m]	0.050	0.003	0.012	NA
	Computation time [s] ^a	16	16	17	34
	Number of processed LiDAR points	13,458,497			
	Coefficient of variation of geoid undulations within the flight strip [%]	0.0696			
	Maximum geoid undulation difference within the flight strip [m]	0.153			
Gador	Mean [m]	-0.005	0.008	-0.002	NA
LiDAR	Maximum signed vertical residual [m]	0.201	-0.015	-0.017	NA
	Standard deviation [m]	0.107	0.004	0.007	NA
	Computation time [s] ^a	12	12	14	17
	Number of processed LiDAR points	6,979,848			
	Coefficient of variation of geoid undulations within the flight strip [%]	0.1893			
	Maximum geoid undulation difference within the flight strip [m]	0.343			

^a Running time to compute all the LiDAR points, including "No simplification approach" (the results have been obtained by using a 2.8 Gz Intel CoreTM 2 Duo 3.48 Gb RAM).

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Fig. 2. Situation of the two tested working areas called Coastal LiDAR and Gador LiDAR. Contour level representation of EGM08-REDNAP geoid undulations (0.10 m contour interval).

undulations obtained from the three simplified approaches and those obtained from the non-simplified case (ground truth). The accuracy results regarding "One known point approach" are significantly worse than those obtained for the approaches "Bilinear interpolation from four boundary points" and "LiDAR trajectory". In fact, although the mean value of errors turned out to be very close to zero (no systematic error) in all the cases, the first simplification tested also showed a higher standard deviation (random error or variability) and maximum signed residual with respect to the others.

Table 1 also depicts some parameters related to the efficiency of each approach with respect to computation time and the number of LiDAR processed points. In this sense, the simplified approaches called "Bilinear interpolation from four boundary points" and "One known point" supposed a very similar average computational burden (close to 711,000 LiDAR points processed per second as the average of the two LiDAR areas) as compared with the one offered by the "No simplification approach", which was able to compute up to 403,000 points per second. It means a significant increase in computational performance, quantitatively close to 76%, meanwhile maintaining a still acceptable accuracy level of around 10.7 cm (measured as standard deviation) and 20.1 cm maximum signed residual in the worst of the cases. It is necessary to underline the excellent performance of the "Bilinear interpolation form four boundary points" approach, which yielded standard deviations clearly lower than 1 cm and maximum absolute residuals below 2 cm. The approach called "LiDAR trajectory" offered a little poorer accuracy results than "Bilinear interpolation from four boundary points", meanwhile showing a lesser efficiency because of its slightly lower computational performance (around an average of 645,000 LiDAR points processed per second).

The observed experimental accuracy has been found to be related to the geoid undulation variability (measured as coefficient of variation; CV) within the flight strip, above all in the case of "One known point" approach, which turned out to be, as it could be expected, the most sensitive of the three tested simplifications with regard to geoid undulation variations (Table 1). In this way, both CV and maximum geoid undulation difference within the flight strip (ΔN) can be considered fairly good indicators to know in which situations would be recommended to apply this type of simplifications and so making possible to extrapolate the obtained results to other potential LiDAR projects.

Fig. 3 presents several graphs which relate the capture time of points (x-axis) to the differences of the undulations obtained by the tested simplifications with respect to the "No simplification approach". In other words, it may be regarded as a residuals distribution along the capture time. As it could be expected, these results showed a linear tendency for the case "One known point" approach (Fig. 3a and d) with a null value in the middle of the time scale. Actually, this is the situation of the point selected in order to obtain the simplified value of undulation for the entire flight strip. Both LiDAR projects provided maximum absolute differences of 12.8 and 20.1 cm. This situation is opposite to that observed for the case "Bilinear interpolation from four boundary points" approach, where lower differences at the initial and final part of the time scale were registered (Fig. 3b and e), whereas the maximum value is found around the middle of the time scale. In any case, the maximum absolute error was lower or equal than 1.5 cm. The case "LiDAR trajectory" approach (Fig. 3c and f) depicted a residuals distribution more spread along the time scale, taking maximum values lower than 2.1 cm. In this last case it is worth noting that a higher variability in differences for a short time interval was observed. This was because of the capture system (sensor rotates transversally to the trajectory). For all cases the results showed that error clearly grows with the distance to the point with known undulation.

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Fig. 3. Results obtained where the horizontal axis represents relative time along one LiDAR strip and the vertical axis indicates error (m). *Note*: A1 = "One known point approach"; A2 = "Bilinear interpolation from four boundary points approach"; A3 = "LiDAR trajectory approach".

Analyzing the distribution of the differences or errors, the "Bilinear interpolation from four boundary points" approach performed slightly better than "LiDAR trajectory" approach, which showed slightly higher differences distributed among the points which were far away from the trajectory. Case "One known point" approach was the simplification which presented the greatest error. For this case the differences were not negligible, taking into account the vertical accuracy currently offered by LiDAR technology, i.e., between 5 and 20 cm when considering flight heights up to 2000 m above the ground (e.g. Beraldin et al., 2010).

Studying the results by working areas, Coastal LiDAR yielded more accurate results than Gador LiDAR due to lower undulation variability within the flight strip (see Fig. 2 and Table 1), which somehow limited the potential uncertainty derived from the application of the tested simplifications. In fact, the coefficient of variation of geoid undulation values within the flight strip was higher in the case of Gador LiDAR (Table 1), which can explain the presence of larger errors when applying the simplification approaches on this working area. It is noteworthy that by restricting the uncertainty (measured as standard deviation) of the conversion from ellipsoid to orthometric heights ($\sigma_{z \text{ conversion}}$) below around 1 cm (approaches A2 and A3 in Table 1), the final vertical accuracy for the set of LiDAR-derived orthometric heights ($\sigma_{z \text{ total}}$) would practically keep the vertical accuracy of the original LiDAR data ($\sigma_{z \text{ LiDAR}}$), as it can be easily checked from Eq. (3).

$$\sigma_{z \text{ total}} = \sqrt{\sigma_{z \text{ conversion}}^2 + \sigma_{z \text{ LiDAR}}^2}$$
(3)

Notice that Eq. (3) is based on general error-propagation theory, assuming that the sources of error are linearly independent or uncorrelated (i.e. the covariance terms between variables have been neglected) and, likewise, that the errors are randomly distributed. It is important to bear in mind the local conditions in which our experimental study has been conducted. In fact, the coefficient of variation of the geoid undulations within the flight strip has presented values below 0.2% in both working areas, whereas the maximum ΔN was computed within Gador LiDAR, taking a value lower than 0.4 m. Thus, both aforementioned values may be regarded as reasonable bounds to apply the proposed simplification

approaches in other LiDAR projects and, meanwhile, approximately maintaining their corresponding accuracy figures. Obviously, the dimensions of the flight strip have to be restricted to keep those values not much higher than those reported in this work. Thus, and taking into account the last consideration, it would be strongly recommended to apply the approach called "Bilinear interpolation from four boundary points" due to its low computational burden and outstanding accuracy figures. Furthermore, this approach can be easily implemented even by using widely known spreadsheet programs (e.g. Microsoft ExcelTM).

Finally, if the required working area was covered by several LiDAR strips and it was needed to transform the ellipsoid heights by blocks or strips, what is a very common practice, it should be taken into account the geometric continuity of overlapping zones in order to maintain the stability of undulation values into space (Vosselman and Maas, 2010). In this case, the "Bilinear interpolation from four boundary points" approach would also be the most suitable choice because the transformation errors would be lower along the borders of the LiDAR flight strip.

4. Conclusions

The results of this study clearly point out different error patterns depending on the simplification used. Through the presented analysis, the values and distribution of errors for each case have been established. Under the operational conditions in which this work has been carried out (i.e., coefficient of variation of geoid undulations within the flight strip < 0.2%), it may be strongly recommended the simplification approach called "Bilinear interpolation from four boundary points" to efficiently transform LiDAR-derived ellipsoid heights to orthometric ones without a significant loss of accuracy. Furthermore, this last approach can be easily implemented in the widely known and easy-to-use spreadsheet tools.

Briefly, along this work we have tried to demonstrate that there is no need to be afraid about transforming ellipsoid heights to orthometric ones. In fact, sometimes this process is supposed to contribute as a significant source of vertical error, being necessary to use sophisticated methods to assure the most accurate transformation. It has been proved that, under certain conditions related to geoid undulation variability within the flight strip, this is not actually true.

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References

- Aguilar, F.J., Mills, J.P., 2008. Accuracy assessment of LiDAR-derived Digital Elevation Models. The Photogrammetric Record 23 (122), 148–169.
- Aguilar, F.J., Mills, J.P., Delgado, J., Aguilar, M.A., Negreiros, J.G., Pérez, J.L., 2010. Modelling vertical error in LiDAR-derived digital elevation models. ISPRS Journal of Photogrammetry and Remote Sensing 65 (1), 103–110.
- Beraldin, J.A., Blais, F., Lohr, W., 2010. Laser scanning technology. In: Vosselman, G., Maas, H.-G. (Eds.), Airborne and Terrestrial Laser Scanning. Whittles Publishing, Dunbeath, Scotland, UK, pp. 1–42.
- BOE: Boletín Oficial del Estado de España, 2007. Real Decreto 1071/2007, de 27 de julio, por el que se regula el sistema geodésico de referencia oficial

en España. Boletín Oficial del Estado. http://www.boe.es/boe/dias/2007/08/29/pdfs/A35986-35989.pdf (accessed 28 January 2011).

- Leica Geosystems, 2008. Leica ALS60 Airborne Laser Scanner Product Specifications. http://www.leica-geosystems.com/downloads123/zz/airborne/als60/productspecification/ALS60_ProductSpecs_en.pdf (accessed 4 February 2011).
- Flood, M., 2004. ASPRS guidelines. Vertical accuracy reporting for LiDAR data. http:// www.asprs.org/society/committees/standards/Vertical_Accuracy_Reporting_ for_Lidar_Data.pdf (accessed 4 February 2011).
- Hodgson, M.E., Bresnahan, P., 2004. Accuracy of airborne Lidar-derived elevation: empirical assessment and error budget. Photogrammetric Engineering & Remote Sensing 70 (3), 331–339.
- Hooijberg, M., 1997. Practical Geodesy. Springer-Verlag, Berlin.
- IGN (Instituto Geográfico Nacional de España), 2009. El nuevo modelo de geoide para España EGM08 – REDNAP, 2009. ftp://ftp.geodesia.ign.es/documentos/ EL%20NUEVO%20MODELO%20DE%20GEOIDE%20PARA%20ESPA_A%20EGM08-REDNAP.pdf (accessed 28 January 2011).
- IGN (Instituto Geográfico Nacional de España), 2010. Programa de Aplicaciones Geodésica PAG. ftp://ftp.geodesia.ign.es/utilidades/PAG/ (accessed 12 March 2011).
- Ministerio de Fomento de España, 2010. Especificaciones Técnicas para VUELO LiDAR y procesado del MDE. Plan Nacional de Ortofotografía Aérea. http://www.mfom.es/ (accessed 12 March 2011).
- Pavlis, N.K., Holmes, S.A., Kenyon, S.C., Factor, J.K., 2008. An Earth Gravitational Model to Degree 2160: EGM2008, Geophysical Research Abstracts, 10.
- Smith, S.L., Holland, D.A., Longley, P.A., 2003. The effect of changing grid in the creation of laser scanner digital surface models. In: Proceedings of the 7th International Conference on GeoComputation, September 8th–10th, University of Southampton (UK), pp. 1–15.

Torge, W., 2001. Geodesy, 3rd ed. W. de Gruyter, Berlin/New York.

Vosselman, G., Maas, H.-G., 2010. Airborne and Terrestrial Laser Scanning. Whittles Publishing, Dunbeath, Scotland, UK.