Optimal Provision of Public Inputs in a Second Best Scenario*

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27 January, 2006

Abstract

This paper studies the optimal level of public inputs under two different tax settings: with lump-sum taxes and with taxes on labour. A numerical simulation is carried out to compute the level of public spending in each scenario. Using the methodology proposed by Gronberg and Liu (2001), we obtain that the level of public input provided under the second best scenario is higher than that corresponding to the first best outcome. The effect of changes to some parameters on the level of public input is also studied.

JEL Classification: H21, H3, H41, H43

Keywords: Second best, excess burden.

*Parts of this paper have been presented in seminars at centrA-Seville, the 60th Congress of IIPF, the XII Encuentro de Economía Pública, the XI Encontro de Novos Investigadores de Análise Económico and the 3rd IASC World Conference on Computational Statistics & Data Analysis. We are grateful to T. Aronsson, G. Fernandez de Cordoba, M. Matsumoto, M.D. Morales and J. Rodero for their comments and suggestions. The usual disclaimer applies. D. Martinez thanks the hospitality of Umea University and of Fiscal Policies Division of the ECB, where parts of this paper were done. Both authors acknowledge the financial support by the Spanish Institute for Fiscal Studies. Corresponding author: D. Martinez. Centro de Estudios Andaluces. C/ Bailén, 50. 41001 Seville. Spain. Phone: 34 955 055 217. E-mail: diego.martinez@centrodeestudiosandaluces.es

1 Introduction

Part of the current debate regarding public goods provision deals with the optimal level of public goods. Indeed, the controversy concerns more the quantity of public goods than the optimality rules derived from the first order conditions. Papers such as Wilson (1991), Chang (2000) and Gaube (2000) highlight this topic, in many cases using numerical examples (and counterexamples). The underlying idea of these papers is that using distortionary taxation leads to an optimal level of public goods below its first-best level; this is based on the argument that the optimal extent of public spending should be inversely related to the welfare cost derived from distorting taxation. However, Gaube (2000) shows that this statement is not as straightforward as it might seem.

All these issues have received very little attention in terms of public inputs. However, we believe that the particular features of productive public spending deserve a specific treatment, as Feehan and Matsumoto (2000, 2002) has recently shown. In this paper, we use a simple model where public spending yields productive services to firms. Two different tax settings are available for government: a lump-sum tax and a tax on labour. A numerical simulation is carried out to compute the levels of public spending.

In order to discuss on the level of public spending provision, we follow the approach suggested by Gronberg and Liu (2001), which is based on the sign of the marginal excess burden. Given the assumptions of our model, we cannot determine with certainty whether the first-best level will overcome the second-best level or not. In fact, the numerical simulation indicates that the second best level exceeds the first-best level with the three utility functions used, being closely linked to the extent of profits. Other results concerning the impact of changes in the output elasticity with respect to public input and the number of households are also provided.

The structure of the paper is as follows. Section 2 presents the model. Section 3 discusses the application of Gronberg and Liu's methodology to our case and presents the main results. Finally, section 4 concludes.

2 The model

We assume an economy of n identical households whose utility function[1] is expressed as $u(x,h)$ where x is a private good used as a numeraire and h the leisure. Let Y be the total endowment of time such that $l = Y - h$ is the labor supply. Output in the economy is produced using labour services and a public input g according to the aggregate production function $F(nl,g)$ .This function satisfies the usual assumptions: increasing in its arguments and strictly concave. Constant returns to scale are assumed in all factors, including the public input. Using the Feehan and Matsumoto's (2002) nomenklature, the public input is treated as a firm-augmenting production factor which creates rents. Output can be used costlessly as x or g.

Labour market is perfectly competitive so that the wage rate is linked to the marginal productivity of labour:

$$\omega = F_L(nl,g), \tag{1}$$

where firms take g as given.Profits can be defined as:

$$\pi = F(nl,g) - nl\omega \tag{2}$$

We distinguish two different tax settings. Firstly, we consider a lump-sum tax T so that the representative household faces the following problem:

$$\begin{aligned} & Max \quad u(x,h) \\ s.t. \quad : \quad & x = \omega(Y-h) - T, \end{aligned} \tag{3}$$

which yields the labour supply $l(\omega,T)$ and the indirect utility function $V(\omega,T)$. It is to be assumed that $l_\omega \geq 0$.

For later use, we describe the comparative statics of $\omega(g,T)$ and $\pi(g,T)$:

$$\omega_g = \frac{F_{Lg}}{1 - nF_{LL}l_\omega} > 0 \tag{4}$$

$$\omega_T = \frac{nF_{LL}l_T}{1 - nF_{LL}l_\omega} > 0 \tag{5}$$

[1] The properties of $u(x,l)$ are the standard ones to ensure a well-behaved function: strictly monotone, quasiconcave and twice differentiable.

$$\pi_T = -\frac{nlF_{LL}l_T}{1 - nF_{LL}l_{\omega_N}} < 0 \quad (6)$$

The optimization problem of government in the first-best scenario is as follows:

$$\begin{array}{rl} & Max \quad V(\omega, T) \\ s.t. \quad : & g = nT + \pi(\omega, T) \end{array} \quad (7)$$

An alternative scenario is that using a specific tax on labour τ. Under this tax setting squeme, the consumer's optimization problem could be expressed as:

$$\begin{array}{rl} & Max \quad u(x, h) \\ s.t. \quad : & x = (\omega - \tau)(Y - h) \end{array} \quad (8)$$

obtaining $l(\omega_N)$ and $V(\omega_N)$, where $\omega_N = \omega - \tau$ is the net wage[2]. Again for future reference we derive the following results:

$$\omega_\tau = \frac{-nF_{LL}l_{\omega_N}}{1 - nF_{LL}l_{\omega_N}} > 0 \quad (9)$$

$$\pi_g = F_g - (nF_{LL}l_{\omega_N} + 1)\, nlF_{Lg} \gtrless 0 \quad (10)$$

$$\pi_\tau = (1 - \omega_\tau)\, n^2 lF_{LL}l_{\omega_N} < 0 \quad (11)$$

In the second-best scenario, the optimization problem of government is as follows:

$$\begin{array}{rl} & Max \quad V(\omega, \tau) \\ s.t. \quad : & g = n\tau l(\omega_N) + \pi(\omega, \tau) \end{array} \quad (12)$$

On the basis of both tax settings and after some manipulations, an important condition for the optimal provision of public inputs is achieved[3]:

$$F_g = 1 \quad (13)$$

[2] Hereafter, a subscript is used for partial derivatives.

[3] See Martínez and Sánchez (2005) for more details.

It means that the production effects of public input must equal to the its marginal production cost at optimum.

On the basis of optimality rules such as those before, Gaube (2000) and Chang (2000) have suggested several criteria for level comparisons between the first and second-best environments in the case of public goods. The support for their approaches is related to the complementarity or substitutability relationships among private goods, and between these and public goods as well. Unfortunately, this procedure has a limitation in our case: the public input does not enter the utility function as an argument, and consequently cannot be directly defined as a substitute or complement to the (taxed) private goods.

To gain an insight into whether the second-best level may exceed the first-best level, we shall follow the approach suggested by Gronberg and Liu (2001), which is better suited to our environment. The crucial point is the concept of marginal excess burden (MEB). Previously, we define the total excess burden (TEB) of a tax system as the difference between the equivalent variation measure (absolute value) of the loss in utility due to taxation and the revenue collected. Algebraically, TEB can be given implicitly in our case by

$$V(\omega(g,\tau)-\tau, Y-TEB-R)=V(\omega(g,T), Y) \tag{14}$$

or explicitly:

$$TEB=-e(\omega(g), V(\omega(g,\tau)-\tau))+Y-R, \tag{15}$$

where $R=\tau l(\omega(g,\tau)-\tau)+\pi(g,\tau)/n$ is the revenue per capita. Hence, the MEB can be defined as $MEB=\dfrac{dTEB}{dR}$.

Gronberg and Liu (2001) in their Proposition 1 and Proposition 3 claim that if the utility function is strictly quasi-concave and the $MEB>0$ for all R, then the second-best public good level lies below the first-best level (Sufficient condition). Additionally, they highglight the possibility of studying the sign of $\dfrac{dTEB}{d\tau}$ as a way of elucidating the sign of the MEB because $\dfrac{dR}{d\tau}>0$[4].

Operating in (15) we obtain that

[4] Note that $\dfrac{dTEB}{d\tau}=\dfrac{dTEB}{dR}\dfrac{dR}{d\tau}$. Then, given $\dfrac{dR}{d\tau}>0$, we achieve the equality of the

$$\frac{dTEB}{d\tau} = l_c\omega_g g_\tau - l\left(\omega_\tau + \omega_g g_\tau\right) - \tau\frac{dl}{d\omega_N}\left(\omega_\tau - 1 + \omega_g g_\tau\right) - \frac{\pi_\tau + \pi_g g_\tau}{n} \quad (16)$$

Next we study some particular utility functions to provide some insights about the level comparisons in the provision of public inputs.

3 Level comparisons of public input provision

We consider three different utility functions in an attempt to achieve results as general as possible. Particularly, we have chosen the quasi-linear utility function (Gronberg and Liu, 2001); the Cobb-Douglas utility functions (Atkinson and Stern, 1974; Wilson, 1991); and the CES utility function (Wilson, 1991b; Gaube, 2000). Specifically,

$$U(x,h) = x + 2h^{\frac{1}{2}} \quad (17)$$

$$U(x,h) = a\log x + (1-a)\log h \quad (18)$$

$$U(x,h) = \left(x^\rho + h^\rho\right)^{\frac{1}{\rho}}, \quad (19)$$

where $a \in (0,1)$ and $\rho = 0.5$. In addition, we assume a Cobb-Douglas production function given by $F(nl,g) = (nl)^\alpha g^{1-\alpha}$ where $\alpha \in (0,1)$.

We obtain the specific expressions of (16) in the different cases. The general conclusion is that the sign of this expression is indeterminated, so it could be possible a level reversal taking as a reference the paper by Gronberg and Liu (2001). As an example, we show here the analysis of the sign of the MEB for a Cobb-Douglas utility function:

$$\frac{dTEB}{d\tau} = \left(l_c + nF_{LL}l_{\omega_N}l\right)F_{Lg}\frac{dg}{d\tau} - \frac{1}{n}F_g\frac{dg}{d\tau} \gtrless 0,$$

where some simplifications have been done[5]. A further analysis would lead to study how the MEB is affected when other variables such as population

sign of the rest expressions. That is, $sign\left(MEB\right) = sign\left(\frac{dTEB}{d\tau}\right)$

[5]Note that $\frac{\partial l}{\partial \omega_N} = \omega_\tau = \pi_\tau = 0$.

or the parameters of the production function change[6]. When the others two utility functions are used, the ambiguity in sign remains[7].

In the numerical simulation we perform different values for each parameter involved in the model have been used. We consider $a \in \{0.1, \mathit{0.5}, 0.9\}$, $\alpha \in \{0.6, \mathit{0.7}, 0.8\}$ and $n \in \{1, \mathit{100}, 1000\}$. The values relative to our benchmark are emphasized.

Next, the results are obtained by solving the lump-sum problem -equations (3) and (7)- and the distortionary taxation problem -equations (8) and (12). The government budget constraint is also used in both non-linear equation systems which has been solved using the Newton-Raphson's well-known method. Routines used in the simulations are available upon request.

Tables 1-3 show the main results. Public input provided under the second-best tax setting is always higher than that corresponding to the first-best environment. In principle, this is contrary to the bulk of existing literature, which generally states that the first-best level exceeds the second-best level. When the results are explored more extensively one realizes that the key variable determining the level of public input is profits. Whereas the values of tax rates under both environments are very low[8], almost the whole public input is financed on the basis of economic profits fully taxed by the government. This is in line with the characterization of the public input as firm-augmenting. Recall that the public input is said to be firm-augmenting when the production function exhibits constant returns to scale in all factors, including the public input. In this case, profits arise and this can be interpreted as an externality. As long as the government becomes the owner of these profits, the positive effect of public inputs is taken into account, thus the externality is internalized. In other words, since all profits accrue to government, the optimal quantity of public input is directly related to the extent of these profits.

Our simulation allows additional results to be obtained regarding the sensitivity to changes in parameters of the utility and in production function and in the number of households. Firstly, in the case of the Cobb-Douglas utility function (Table 2), it can be seen how the public input level increases when the preference for the private good goes up (parameter a). The higher

[6]For example, we have checked that when n raises, MEB increases. However, if we increase F_g, MEB becomes lower. But a deeper analysis has to be implemented here.

[7]Details are availaible upon request.

[8]We consider a minimum value of tax rates along the simulation in order to have a significant value which let us distinguish between both tax settings.

the preference for the private good, the smaller the preference for leisure, and consequently more time is devoted to work. Given the assumptions of the model, this means to increase the production, and hence more profits[9].

Secondly, the output elasticity to public input $(1-\alpha)$ has a positive relationship with the public input level. In some sense, this elasticity measures the extent of the externality deriving from public input provision. The size of this effect, which is completely internalized by the government when all rents are taxed, determines the level of public input, and a direct relationship between $1-\alpha$ and g is to be expected.

Thirdly, there is a direct link between the number of households and the level of public inputs. The proportion by which this occurs is linear, that is, if population increases by 10 times, the level of public input goes up by the same proportion. Again, profits define the optimal provision of public spending. On the basis of expression (2), and since the production function exhibits constant return to scale (i. e., it is homogeneous of degree 1), it can be claimed that the function $\pi(.)$ is homogeneous of degree 1. Accordingly, increases in the number of households are followed by increases in profits at the same rate, and consequently by identical increases in the public input.

4 Concluding remarks

This paper has dealt with an issue to which the existing literature has not paid much attention: the optimal *level* of public inputs under different tax settings. Previous contributions have focused on the case of consumption public goods or have discussed the optimal *rules* of productive public spending. However, both the social welfare implications of taxation and the characterization of public inputs as growth-enhacing public instruments make this issue highly relevant for policy-makers.

We have built a simple model where public inputs provide productive services to firms. Two different tax settings have been considered: one with lump-sum taxation and another using a specific tax on labour. We have shed some light on the controversy concerning level comparisons between both tax

[9] Formally, $l(\omega_N) = Ya \Longrightarrow \dfrac{\partial \pi}{\partial a} = -nYa\dfrac{\partial \omega}{\partial a} \Longrightarrow sign(\dfrac{\partial \pi}{\partial a}) \neq sign(\dfrac{\partial \omega}{\partial a})$
From (1), $\dfrac{\partial \omega}{\partial a} = \alpha(1-\alpha)\left(\frac{g}{n}\right)^{1-\alpha}(Ya)^{\alpha}\frac{-1}{Ya^2} < 0$. Then, $\dfrac{\partial \pi}{\partial a} > 0$

settings. Contrary to the existing literature on public goods, which generally holds that the first-best level is higher than the second-best level, the relationship is the reverse for the case of public inputs. This result is a consequence of the extent of profits, which are used in our framework to internalize the externality of the public input. Moreover, we have detected a positive relationship between the level of public input and its output elasticity in the production function, and with the number of households in the economy.

This paper raises various policy implications. Firstly, optimality rules and levels in the public input provision require a different treatment to those corresponding to public goods. This is due to the more intense feedback effect derived from public inputs and certain characteristics of the externalities involved in their provision. The second policy implication is precisely related to th nature of this externality. As long as the public input provision may generate external effects, benefit-based taxation becomes an efficient source of resources for the government and a means of improving efficiency.

Further research on this issue is warranted. Gronberg and Liu's criterion for level comparisons could be studied in a more detailed way for the case of public inputs. Similarly, it would be interesting to carry out a further research into the behavior of the MEB with public inputs, and according to certain parameters of the model, namely the number of households, output elasticity to the public input, and so on.

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Tables

Table 1: Quasi-linear utility.

	Benchmark	$n = 100$		$\alpha = 0.7$	
		$\alpha = 0.6$	$\alpha = 0.8$	$n = 1$	$n = 1000$
Lump-sum					
Public Input	328.9051	319.0194	275.6000	3.2891	3289.0510
Utility	12.4214	10.8875	14.7090	12.4214	12.4214
Labour Supply	18.2872	14.6143	20.5125	18.2872	18.2872
Total Production	1093.0100	795.0487	1373.0000	10.9301	10930.1000
Profits	327.9029	318.0195	274.6000	3.2790	3279.0290
Distorsionary					
Public Input	352.8207	339.9574	299.5921	3.5282	3258.2070
Utility	12.4146	10.8823	14.7001	12.4146	12.4146
Labour Supply	18.2620	14.5542	20.5005	18.2620	18.2620
Total Production	1115.1930	813.5083	1395.4580	11.1519	11151.9300
Profits	334.5578	325.4033	279.0916	3.3456	3345.5780

Source: Benchmark $(n = 100, \alpha = 0.7)$

Table 2: Cobb-Douglas utility.

	Benchmark	$n = 100, \alpha = 0.7$		$n = 100, a = 0.5$		$a = 0.5, \alpha = 0.7$	
		$a = 0.1$	$a = 0.9$	$\alpha = 0.6$	$\alpha = 0.8$	$n = 1$	$n = 1000$
Lump-sum							
Public Input	216.5287	44.7812	388.2684	262.5820	161.8744	2.1653	2165.2870
Utility	2.0486	2.7657	2.0676	1.9241	2.1722	2.0486	2.0486
Labour Supply	12.0119	2.4213	21.6024	12.0153	12.0093	12.0119	12.0119
Total Production	718.4291	145.9375	1290.8950	653.9537	804.3604	7.1843	7184.2910
Profits	215.5287	43.7813	387.2685	261.5815	160.8721	2.1553	2155.2870
Distorsionary							
Public Input	231.8385	46.3677	417.3092	280.2976	175.3683	2.3184	2318.3850
Utility	2.0481	2.7656	2.0668	1.9235	2.1717	2.0481	2.0481
Labour Supply	12.0000	2.4000	21.6000	12.0000	12.0000	12.0000	12.0000
Total Production	732.7950	146.5590	1319.0310	670.7435	816.8367	7.3280	7327.9500
Profits	219.8385	43.9677	395.7093	268.2974	163.3673	2.1984	2198.3850

Source: Benchmark $(n = 100, \alpha = 0.7, a = 0.5)$

Table 3: CES utility ($\rho = 0.5$).

	Benchmark	$n = 100$		$\alpha = 0.7$	
		$\alpha = 0.6$	$\alpha = 0.8$	$n = 1$	$n = 1000$
Lump-sum					
Public Input	128.6810	130.7123	113.4492	1.2868	1286.8100
Utility	34.0278	31.8172	36.8396	34.0278	34.0278
Labour Supply	7.1063	5.9426	8.3889	7.1063	7.1063
Total Production	425.6034	324.2753	562.2460	4.2560	4256.0340
Profits	127.6810	129.7101	112.4492	1.2768	1276.8100
Distorsionary					
Public Input	136.5596	137.6260	122.1769	1.3656	1365.5960
Utility	34.0191	31.8090	36.8294	34.0191	34.0191
Labour Supply	7.0684	5.8919	8.3603	7.0684	7.0684
Total Production	431.6376	329.3317	569.0830	4.3164	4316.3760
Profits	129.4913	131.7327	113.8166	1.2949	1294.9130

Source: Benchmark ($n = 100, \alpha = 0.7$)