Fiscal federalism and public inputs provision: vertical externalities matter

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Abstract

This paper studies the provision of public inputs in a federal system. A vertical tax externality is also considered in a simple general equilibrium model used to analyze the efficiency of equilibria under different scenarios. The results show that the state provision of public inputs may affect ambiguously federal tax revenues, depending on the vertical tax externality, amongst others issues. Moreover, it is proved that achieving a second best allocation is not straightforward for a federal government that plays as Stackelberg leader. At this point, the state's reaction function becomes crucial when the design of vertical grants is restricted.

Keywords: Fiscal federalism, vertical externality, productive public spending.

JEL Classification: H2, H4, H7

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1 Introduction

Traditionally, the study of vertical externalities in a federation has been focused on tax externalities, in which different levels of government share the same tax base. As is well-known, it leads to an overprovision of public goods as long as the deadweight loss of distorting taxation is underestimated by governments. Flowers (1988) deals with this issue through a Leviathan's approach and shows how the federation may end up at the downward-sloping part of the Laffer curve. Papers such as Dahlby and Wilson (1994), Boadway and Keen (1996), and Sato (2000) find similar conclusions when a benevolent government is involved[1]. Moreover, these contributions propose different systems of vertical transfers that correct these externalities between governments.

Vertical externalities may also arise when other aspects are regarded. Boadway at al. (1998) use a model with heterogeneous and partially-mobile agents to make explicit the trend of the states to be too progressive. In terms of interregional trade, Lucas (2004) has shown recently how a federal government as Stackelberg leader can replicate the unitary nation optimum through matching grants in a federation with vertical and horizontal externalities.

An issue upon which the main branch of the literature has not paid much attention is the vertical externality coming from the provision of public inputs. This point refers to the positive or negative effects that the productive public spending by one level of government may exert on other levels' revenues. In addition, this fact can be found in supranational structures such as European Union, in which an important share of its budget is devoted to regional policies based on the provision of infrastructures; there are no doubts that these types of policies have a positive impact on local, regional and federal budget constraints in many Member States.

Anyway, some papers have dealt with this concern. Dalhby (1996) describes the effects of expenditure externalities in a federation, and defines a general framework for matching grants in order to eliminate them. Wrede (2000) deals with productivity increasing public services in a federation consisting of Leviathan governments. Recently, Dalhby and Wilson (2003) examine a model in which state governments provide a productivity-enhancing

[1] Nevertheless, Keen (1998) claims that the effects of federal taxes on state taxes are not so much straightforward as it might seem: under certain conditions, increases in the federal tax rate may reduce the state tax rates. Empirical evidence is miscellaneous (see, for instance, Esteller-More and Sole-Olle, 2001, and Anderson et al., 2004).

public input; they conclude that this externality may have an ambiguous impact on federal revenues, and a matching grant from the federal government to the states is able to correct it.

This paper uses Boadway and Keen's (1996) model to study the efficiency of equilibria when a public input is provided by state governments. We here consider the positive impact of a public input on wage rate through a higher labor productivity. Federal and state governments use per unit taxes on labor instead of ad valorem taxes used by Dahlby and Wilson; it allows us to focus on the (likely) positive externality derived from the public input, rather than other positive vertical externalities that may arise when ad valorem taxes are involved. The behavior of governments has been modeled under different scenarios: as a central government in an unitary country, different governments as Nash competitors, and one level of government (the federal one) acting as Stackelberg leader. Moreover, we wonder about the capability of the federal government to achieve a second-best solution. At this point, we deal with restrictions by employing policy instruments: federal government cannot make use of vertical grants to correct vertical externalities. This way, the paper tries to reproduce a common feature in real federations, namely, constitutional arrangements may prevent the design of intergovernmental transfers based on efficiency criteria exclusively[2].

The results show that, as Dahlby and Wilson (2003) point out, the marginal cost of providing a public input may be under or overestimated in a federal system. However, contrary to Dahlby and Wilson (2003), our paper finds a bias between the unitary and the federal solution, which is not independent of the vertical tax externality. The reasoning followed in this paper sharply contrasts to that of Dahlby and Wilson (2003) because we detect that production efficiency condition does not perform properly as criterion for assessing optimality in federal countries, as they do. Moreover, since no vertical transfers are available in our model, it is not straightforward the ability of federal government behaving as Stackelberg leader to replicate the second best outcome. This paper demonstrates that when the set of policy instruments is restricted, the effectiveness of the federal tax rate to implement the second best optimum depends on the state governments' reaction to changes in federal taxes. Also we obtain that the optimum federal

[2] In addition, this point also allows to relate our model to literature on optimal taxation and the availability of policy tools (see Stiglitz and Dasgupta (1971) and subsequent papers)

tax rate can be positive, unlike Boadway and Keen's (1996) findings.

The structure of the paper is as follows. Section 2 describes the main features of the model. Section 3 provides the second best outcome achieved in an unitary country. Next section compares this result to those reached when federal and state governments play Nash. Section 5 studies whether federal government behaving as Stackelberg leader is able to replicate the second best allocation. Finally, section 6 concludes.

2 The model

We assume a country with a federal government and k identical states. It will allow us to address symmetric allocations and to eliminate the possibility of horizontal grants used for redistribution aims. Due to this symmetry, we will focus our discussion on the bilateral relationship between the federal government and only one state government.

Each state is populated by n identical households that are assumed to be completely immobile[3]. Household's utility function is given by the separable form:

$$u(x, l) + B(G), \tag{1}$$

where x is a private good used as numeraire, l is the labor supplied, and G is a pure public good provided by the federal government. The properties of the function $u(x, l)$ are the standard ones, and $B(G)$ is increasing and concave. The representative household faces the following budget constraint:

$$x = (\omega - \tau) l, \tag{2}$$

where ω is the wage rate and τ the per unit tax on labor. Household's optimization problem consists of maximizing (1) subject to (2), and that yields labor supply $l(\omega - \tau)$ and indirect utility function $V(\omega - \tau) + B(G)$.

[3]Relaxing the assumption of complete household immobility would have no effects on the efficiency of the equilibria and governments' behavior, as long as states are assumed to be symmetric (Proposition 4 in Boadway and Keen, 1996). By contrast, when there is (perfect o imperfect) inter-regional population movements and states are not homogeneous, the second best allocation does not require the equalization of the conventional form of the marginal cost of the public funds across regions and layers of government (Sato, 2000).

It is assumed that $l^{'} > 0$[4].

Output in the economy is produced using labor services and the public input g according to the following aggregate state production function:

$$F(L, g), \tag{3}$$

where $L = nl$. This function satisfies the usual assumptions: increasing in its arguments and strictly concave. Output can be used costlessly as x, G or g. Labor market is perfectly competitive so that we can write:

$$\omega = F_L [nl(\omega - \tau), g] \tag{4}$$

It allows us to achieve the wage function $\omega(g, \tau, n)$. Some results of comparative statics can be found now; they will be used later:

$$\omega_g = \frac{F_{Lg}}{1 - F_{LL} n l^{'}} > 0 \tag{5}$$

$$\omega_\tau = \frac{-F_{LL} n l^{'}}{1 - F_{LL} n l^{'}} > 0 \tag{6}$$

Economic profit is defined as a residual, or

$$\pi(g, \tau, n) = F[nl(\omega(g, \tau, n) - \tau), g] - nl[\omega(g, \tau, n) - \tau]\,\omega(g, \tau, n) \tag{7}$$

Again, it is useful to obtain some results for later use:

$$\pi_g = F_g - \left(F_{LL} n l^{'} \omega_g + F_{Lg}\right) nl \lesseqgtr 0 \tag{8}$$

$$\pi_\tau = (1 - \omega_\tau) F_{LL} n^2 l l^{'} < 0 \tag{9}$$

Note that the effect of public inputs on rents is ambiguous because g increases output (and hence, the economic profits) but also exerts a positive impact upon wage rate, reducing rents.

Each level of government sets its own tax rate on labor. Denoting T as the tax rate established by federal government and t as the corresponding variable at state level, it can be written $\tau = T + t$. Thus, the revenue raised by federal government to finance G is:

[4] Hereafter, differentiation is denoted by a prime for functions of a single variable, while a subscript is used for partial derivatives.

$$G\left(T,t,\theta,g,n,S\right)=knTl\left(\omega\left(g,\tau,n\right)-\tau\right)+k\theta\pi\left(g,\tau,n\right)-kS, \tag{10}$$

where $0\leq\theta\leq1$ is the proportional tax rate on profits levied by federal government, and S is a vertical transfer between both levels of government[5]. Throughout this paper, θ is assumed to be fixed and exogenously determined. The effects of changes in T, t, g and S on federal budget constraint are given by:

$$G_T=\left(\omega_\tau-1\right)knTl'+knl+k\theta\pi_\tau \tag{11}$$

$$G_t=\left(\omega_\tau-1\right)knTl'+k\theta\pi_\tau=G_T-knl \tag{12}$$

$$G_g=knTl'\omega_g+k\theta\pi_g \tag{13}$$

$$G_S=-k \tag{14}$$

The state revenue constraint is

$$g\left(t,T,\theta,n,S\right)=ntl\left(\omega\left(g,\tau,n\right)-\tau\right)+\left(1-\theta\right)\pi\left(g,\tau,n\right)+S \tag{15}$$

State government appears as the only agent providing the public input. Note that all economic profits are taxed away by both levels of governments because rents are efficient resources for public sector[6]. For future reference, the impacts of changes in t, T and S are obtained:

$$g_t=\left(\omega_\tau-1\right)ntl'+nl+\left(1-\theta\right)\pi_\tau \tag{16}$$

$$g_T=\left(\omega_\tau-1\right)ntl'+\left(1-\theta\right)\pi_\tau=g_t-nl \tag{17}$$

$$g_S=1 \tag{18}$$

[5] S may have either sign and it is defined as a lump-sum grant in the sense of Boadway and Keen (1996) or Sato (2000).

[6] We establish here that the country is under-populated in order to avoid that tax on rents may suffice to finance the first-best level of public good (Wildasin, 1986).

When one of the equations (12), (13) or (17) is different to zero a vertical externality arises. Equations (12)-(13) show how federal government's tax revenues are affected by fiscal decisions taken by state government on the tax rate and on the provision of the public input, respectively, while equation (17) is the effect of the federal tax upon state government's revenues.

3 The second-best allocation in an unitary country

Characterizing a vertical externality requires to consider the differences between the optimal solution in an unitary country, and the solution achieved when several levels of government exist. In this section, we obtain the first order conditions for the optimal provision of the national public good G and the public input g in an unitary country.

The central government chooses the values of G, g and τ to maximize the representative household's utility subject to an aggregated budget constraint[7]. Formally,

$$\begin{aligned} & Max \quad V(\omega-\tau)+B(G) \\ s.t. \quad : \quad & G+kg=kn\tau l(\omega(g,\tau,n)-\tau)+k\pi(g,\tau,n), \end{aligned} \tag{19}$$

First order conditions for G, g and τ are, respectively, as follows:

$$B^{'}(G)-\mu = 0 \tag{20}$$

$$V^{'}\omega_g-\mu k+\mu kn\tau l^{'}\omega_g+\mu k\pi_g = 0 \tag{21}$$

$$(\omega_\tau-1)V^{'}+\mu knl+\mu(\omega_\tau-1)kn\tau l^{'}+\mu k\pi_\tau = 0, \tag{22}$$

where μ is the Lagrange's multiplier. Combining (20) with (22), using Roy's identity and the expressions (6) and (9), yield the necessary condition for the second best provision of national public good G[8]:

[7]Wildasin (1986) demonstrates that it is relevant to distinguish between to maximize the per capita utility or the total utility. As cited by Mansoorian and Myers (1995), considering the total utility of households as objective function implies that each state authority has a preference for the population size. With symmetric equilibria and no migration as here, this point is not very relevant, but it would prevent the extension of the results obtained to an environment in which households mobility is allowed. See footnote 3.

[8]Identifying the optima achieved as second best solutions is related uniquely to the presence of distorting labor taxation.

$$\frac{nkB^{'}(G)}{\lambda} = \frac{1}{1-\frac{\tau l^{'}}{l}}, \tag{23}$$

where λ is the private marginal utility of income. LHS of equation (23) is the sum of marginal benefits received by all households living in the federation. RHS of equation (23) is the marginal cost of providing G. As is well-known, this expression is the Samuelson's rule for public good provision corrected by Atkinson and Stern (1974).

After some manipulation with equations (21) and (22), using again Roy's identity, and the expressions (6) and (9), the second best condition for the optimal provision of g can be written as follows:

$$\frac{nV^{'}\omega_g}{\lambda} = \frac{1}{1-\frac{\tau l^{'}}{l}}\left(1 - n\tau l^{'}\omega_g - \pi_g\right) \tag{24}$$

In essence, the interpretation of this equation is the same than before for G. However, it may be worth noting that two terms can be distinguished in the RHS. The first one is the marginal cost of the public funds ($MCPF$); the second one is the tax revenue effect that arises so long as g may affect positively or negatively tax bases through labor productivity and economic profits. Both terms of the RHS define the marginal cost of providing the public input (MCP). Whereas in the case of the consumption public good the $MCPF$ and the MCP are identical, when the public input is considered this distinction is required.

In other words, while the $MCPF$ is a concept that exclusively refers to the use of distorting taxation (regardless providing public goods or inputs), the MCP takes into consideration not only the $MCPF$ but also the complementarity between public spending and government's revenues through what we have named tax revenue effect.

Comparing expressions (23) and (24) a simple result that will be used later is obtained:

Proposition 1 *If central government in an unitary country sets a tax rate* $\tau > 0$ *and* $\pi_g \geqslant 0$*, then the marginal cost of providing* G *will be higher than that corresponding for* g *(Sufficient condition).*

Note that if the provision of g has a positive impact on tax bases, then the marginal cost of providing the public input is below the $MCPF$, even with distorting taxes. In other hand, if Roy's identity is used in the LHS, and expressions (5) and (8) are inserted in (24), manipulation gives:

$$F_g = 1, \tag{25}$$

that is, the production efficiency condition for the provision of public inputs (Diamond and Mirrlees, 1971). It means that the production effects of the public input are equal to its marginal production cost, though distortionary (but optimally set) taxation to be used[9].

4 Vertical externalities when federal and state governments play Nash

The existence of different levels of government may alter the behavior of the agents if they share the same tax base and/or public spending coming from the state governments is able to modify the federal budget constraint.

This section deals with the optimal conditions involved when state and federal governments behave as Nash competitors, that is, each government takes as given the tax rates and the level of public expenditure implemented by other governments. Hence, state's optimization problem consists of selecting the values for g and t in order to maximize the per capita utility of the state, taken its own budget constraint into account. Formally,

$$\begin{array}{rl} & Max \quad V\left(\omega\left(g,\tau,n\right)-\tau\right)+B\left(G\right) \\ s.t. \ : & g=ntl\left(\omega\left(g,\tau,n\right)-\tau\right)+\left(1-\theta\right)\pi\left(g,\tau,n\right)+S \end{array} \tag{26}$$

First order conditions we obtain are:

$$V^{'}\omega_g-\mu+\mu ntl^{'}\omega_g+\mu\left(1-\theta\right)\pi_g = 0 \tag{27}$$

$$\left(\omega_\tau-1\right)V^{'}+\mu nl+\mu\left(\omega_\tau-1\right)ntl^{'}+\mu\left(1-\theta\right)\pi_\tau = 0, \tag{28}$$

The expression that relates marginal benefits and costs of providing the public input at state level can be derived as before:

$$\frac{nV^{'}\omega_g}{\lambda}=\frac{1}{1-\frac{tl'}{l}-\theta F_{LL}nl'}\left(1-ntl^{'}\omega_g-\left(1-\theta\right)\pi_g\right) \tag{29}$$

[9]For further discussion, see Feehan and Matsumoto (2002).

Again, the RHS of equation (29) shows the marginal cost of public input provision when distorting taxes are used and different effects on state tax revenues are involved. A key question arises here about the optimality of this result when comparing to the second best outcome. Our model yields the following proposition:

Proposition 2 *If $T \geq 0$, the MCPF perceived by state governments that play Nash is smaller than the MCPF in an unitary country. However, the marginal cost of providing g perceived by state governments may be higher, equal or smaller than in an unitary country.*

Proof. Using $\tau = T + t$, an alternative expression of the RHS of equation (29) can be obtained:

$$\frac{1}{1 - \frac{\tau l'}{l} + \frac{T l'}{l} - \theta F_{LL} n l'} \left(1 - n\tau l' \omega_g - \pi_g + nT l' \omega_g + \theta \pi_g\right) \qquad (30)$$

First term is the $MCPF$. By assumption, $F_{LL} < 0$ so that denominator is bigger than that of expression (24); thus, the MCPF is smaller with state governments. Regarding the marginal cost of provision, nothing can be said about the magnitude of its second term in relation to (24). Note that by (8), π_g may have either sign. ■

The first part of the proposition is a standard result in the literature, regardless a consumption public good or a public input to be considered. When a vertical tax externality exists, the $MCPF$ for providing both kinds of public expenditures are perceived as lower by state governments. The second part of the proposition pays attention upon the MCP, and claims that the sign of expenditure vertical externality is not determined, so that the state government may under or over-provide the public input.

In this regard, it can be stated that having a positive or negative vertical externality depends firstly on the relative changes in the magnitude of the $MCPF$ and in the tax revenue effect when a federal structure is introduced, and secondly on the sign of the effect of public inputs on rents. In particular, if $\pi_g \geqslant 0$ the sign of both combined vertical externalities will depend on the relative magnitudes of both terms in (30) because they change in opposite senses. By contrast, if $\pi_g < 0$ it may occur that both terms in (30) move in the same sense, and consequently an overprovision of public inputs takes place, but in principle the indetermination is still present.

Some insights on the magnitude of the vertical expenditure externality are provided next. Let $\psi = \frac{MCP_U^g}{MCP_S^g}$ be the ratio between the MCP in an unitary country and the MCP perceived by state governments, both of them referring g (the RHS of equations (24) and (29), respectively). Proposition 2 states that $\psi \gtreqless 1$, i. e., when the state government provides a sub-optimal level of public input, then $\psi < 1$, and otherwise.

Proposition 3 *Ceteris paribus,*
i) ψ is decreasing in the elasticity of wage rate to g if $T > 0$
ii) ψ is decreasing in the marginal productivity of g if $0 < \theta < 1$.
iii) ψ is increasing in the share of rents levied by the federal government θ when $\pi_g \lesseqgtr 0$
iv) ψ is increasing in the elasticity of the labor supply to the federal tax rate T (in absolute value).

Proof. i) Using the terms with ω_g in the second term of (30) -and not present in (24)- and the expression (8) for π_g, yield $nTl'\omega_g + \theta nlF_{LL}l'\omega_g$. Rearranging we can write that $(T - \theta lF_{LL})\, nl'\omega_g > 0$, given (5), $T > 0$ (by assumption) and $F_{LL} < 0$.

ii) Using expression (8) and $0 < \theta < 1$, an increase in F_g reduces the second term of (30). But this effect is bigger in the case of numerator of (24), hence ψ decreases.

iii) Differentiating the RHS of (29) with respect to θ yields $\frac{\left(1-\frac{tl'}{l}-\theta F_{LL}nl'\right)\pi_g-\left(1-ntl'\omega_g-(1-\theta)\pi_g\right)\left(-F_{LL}nl'\right)}{\left(1-\frac{tl'}{l}-\theta F_{LL}nl'\right)^2}$. Since both terms of the RHS of (29) are positive, then $F_{LL} < 0$ and $\pi_g \lesseqgtr 0$ lead to a negative sign in the latter derivative. Thus, MCP_S^g is decreasing in θ, and ψ is increasing in θ.

iv) In the denominator of the $MCPF$ in expression (30), term $\frac{Tl'}{l}$ is the elasticity of labor supply to the federal tax rate T (in absolute value). ∎

In short, the higher the elasticity of wage rate to public inputs and the higher the marginal productivity of public inputs, the more likely is to find infraprovision of public inputs. By contrast, the higher the federal tax rate on rents and the higher the elasticity of labor supply to the federal tax rate, the more likely is to reach overprovision of public inputs.

Parts i) and ii) of proposition 3 show that the sign of vertical expenditure externality depends crucially on the tax revenue effect produced by public input provision. In fact, the more productive the public input, the more tax revenues in both governments. Thus, the gap between what state government sees and what actually happens will be bigger, and it obsviously will lead to infraprovision of public inputs.

Part iii) follows an inverse argument. When public input affects negatively rents, increasing the federal share on economic profits taxes is damaged for federal government, so that risk of overprovision of g rises.

Part iv) of proposition 3 qualifies the statement by Dahlby and Wilson (2003) that vertical tax externalities do not affect public spending externalities. We have found that the extent in which the MCP_S^g differs from the MCP_U^g (i. e., the sign and magnitude of the externality) depends on the tax rate set by federal government or whether labor supply is more o less sensitive to the federal tax rate. It means that both externalities are interrelated[10].

In contrast with that, reasoning followed by Dalhby and Wilson (2003) is based on production efficiency condition and concludes that both externalities are independent each other. Nevertheless, recent papers by Blackorby and Brett (2000) and Kotsogiannis and Makris (2002) have proved that considering production efficiency as criterion for assessing optimality in federal system may be inappropriate. Our model offers a clear insight about that. Using (5) and (8) in expression (29), the following is obtained:

$$\left(\frac{nl\theta F_{Lg}}{F_g} + (1-\theta)\right) F_g = 1 \tag{31}$$

i. e., production efficiency does not hold when governments play Nash. If all the taxes on profits were levied by state government ($\theta = 0$), the above expression would become $F_g = 1$, that is, the efficiency in production of public inputs would be achieved but condition for optimality is not still satisfied (see equation (29) with $\theta = 0$)[11].

[10] In some sense, our vertical expenditure externality holds certain similarities with horizontal externalities. Indeed, assuming a positive impact of state public input on federal tax revenues, it appears a trend towards the infraprovision of g that can be seen as state tax rates being too low (as a result of fiscal competition in the case of horizontal externalities). In such a way, Keen and Kotsogiannis (2002) and Madies (2004) have recently shown the interdependence between both externalities, in line with the results of this paper.

[11] Translating this argument to Dahlby and Wilson's (2003) model, we reach the same conclusion. Using their expressions (6) and (16), an optimal federal tax rate T^* removing both vertical externalities can be achieved (we do something similar in the next section); however, inserting that T^* into their expression (19), production efficiency is not fulfilled. In other words, optimality conditions in federal systems and production efficiency do not necessarily coincide.

5 Federal government plays as Stackelberg leader

Considering that the federal government behaves as Stackelberg leader, anticipating the effects of its actions on the states' decisions, has been the usual way to correct vertical externalities. Federal government sets its tax rate taken as given the states' reaction function, and is able to replicate the second best outcome achieved by government in an unitary country. However, the success of this policy is very sensitive to whether federal government has unrestricted access to vertical transfers or not. As Keen (1998) points out, if vertical transfers are not available for federal government, to achieve the second best allocation is not straightforward, even when the states' reaction function is known.

Our aim here is to shed some light about the capacity of federal government to get the second best outcome when a public input is provided. Vertical transfers will not be allowed for the federal government, whose only instrument to affect the behavior of the states will be the tax rate T. This approach seeks to show not only how the conclusions of the main branch of literature may be modified when policy instruments are restricted, but also to know under which assumptions a federal system with no vertical transfers is able to achieve the second best allocation. This environment also permits dealing with features of real federations, namely, the intergovernmental grants are not usually designed to correct vertical externalities, or sometimes constitutional arrangements prevent the use of vertical transfers based on efficiency criteria exclusively.

As a preliminary point, we should question if there exists a federal tax rate that corrects both vertical externalities. Following Boadway and Keen (1996), we define the marginal vertical externality as follows:

$$\gamma = G_t + G_g, \tag{32}$$

that is, considering the negative and/or positive effects on federal revenues generated by states by means of their own taxes and the provision of public inputs. As at an optimum $\gamma = 0$, inserting (12) and (13) in (32), and solving for T the optimal federal tax rate T^* we find is:

$$T^* = \frac{-\left(\pi_\tau + \pi_g\right)\theta}{\left(\omega_\tau + \omega_g - 1\right) l'n} \lesseqgtr 0 \tag{33}$$

Since there are no vertical transfers between levels of government, federal tax rate T is the unique instrument to offset the two opposite effects that states' decisions have on the federal revenues. The first effect comes from the fact that state tax rates exert a negative impact on federal budget constraint; as pointed out by Boadway and Keen (1996), in that case federal government should subsidy the (common) tax base that, as a result of the tax externality, is over-exploited. But secondly, it is also likely that the provision of public inputs increases federal revenues (positive expenditure externality); thus if t follows T, it may be convenient that a positive federal tax rate to be implemented in order to encourage state taxes. This way, resources for public input provision will rise. Note that in accordance with Proposition 3 (iv), the MCP_S^g is decreasing in $\frac{Tl'}{l}$ (ψ is increasing in $\frac{Tl'}{l}$), so T may stimulate the spending in g.

Another key question is to know what is the state's reaction function with respect to the federal tax rate. So far, each level of government acted independently; under the new framework, federal government knows the effects of its policy on state's behavior, that is, it knows the state's reaction function. As the first order condition for the state's problem can be written as follows

$$V^{'}\omega_g g_t + (\omega_\tau - 1) V^{'} = 0, \tag{34}$$

differentiating this expression with respect to T we obtain:

$$\begin{aligned}(\omega_\tau - 1)(1 + t_T) V^{''}\omega_g g_t + (1 + t_T) V^{'}\omega_{g\tau} g_t + V^{'}\omega_g g_{tt} t_T + V^{'}\omega_g g_{tT} + \\ (\omega_\tau - 1)^2 (1 + t_T) V^{''} + V^{'}\omega_{\tau\tau}(1 + t_T) = 0\end{aligned}$$

As $g_{tT} = g_{tt} + (\omega_\tau - 1) l^{'} n$, rearranging terms and solving for t_T, the above equation can be rewritten as follows:

$$t_T = \frac{-(\omega_\tau - 1) V^{'}\omega_g n l^{'}}{(\omega_\tau - 1) V^{''}\omega_g g_t + V^{'}\omega_{g\tau} g_t + V^{'}\omega_g g_{tt} + (\omega_\tau - 1)^2 V^{''} + V^{'}\omega_{\tau\tau}} - 1 \tag{35}$$

i. e., the state's reaction function. Given the assumptions of our model, nothing can be said about the sign of t_T ($t_T \lesseqgtr 0$). In other words, state tax rates may react ambiguously to changes in the federal tax rate.

Even regarding a more general approach, the doubts about the effects of changes in federal taxes on the national tax rate of the federation remain: the sign of 1+ t_T continues being indeterminate[12]. This ambiguity comes from the unclear net effect of the two vertical externalities when they are considered jointly. While in the case of Boadway and Keen (1996) there exists a remarkable tendency towards overprovision (and the subsequent increase in all tax rates), infraprovision of public inputs (or what is the same, state tax rate being too low) can be found when expenditures externalities are regarded. Even it may lead to reduce the national tax rate τ when federal government sets higher tax rates.

In order to consider how is the response of the state tax rate to changes in the lump-sum transfer, expression (34) is differentiated with respect to S to write:

$$(\omega_\tau - 1) V'' \omega_g g_t t_S + V' \omega_g g_{tt} t_S + (\omega_\tau - 1)^2 V'' t_S = 0, \tag{36}$$

that leads to $t_S = 0$, that is, the tax rate is unaffected by the transfer[13]. Contrary to Boadway and Keen (1996), where this situation is caused by a linear utility function in G, our model does not recognize any ability of the vertical transfer for influencing t, regardless the properties of the utility function. It means that income effects go entirely to the provision of the state public input. Moreover, this is consistent with the null role played by vertical transfers as policy instruments in our model.

At this point, the federal's optimization problem we have to solve is the following:

$$\begin{aligned} & Max \quad V\left(\omega\left(g\left(t,T,\theta,S\right),\tau,n\right)-\tau\right)+B\left(G\left(T,t,\theta,S,g\left(t,T,\theta,S\right)\right)\right) \\ s.t. \quad : \quad & t = t\left(T,\theta,S\right) \end{aligned} \tag{37}$$

As can be seen, both objective function and federal constraint take into consideration the behavior of the states and the influence of federal decisions on them. In such a way, federal government chooses T regarding the first order conditions obtained for state government. Formally:

$$\left[\left(g_t t_T + g_T\right)\omega_g + \left(\omega_\tau - 1\right)\left(1 + t_T\right)\right] V' + B' \left[G_T + G_t t_T + G_g g_T\right] = 0 \tag{38}$$

[12] Note that $1 + t_T = \frac{d\tau}{dT}$.

[13] This result is based on the assumptions of the model after some manipulation in (36). Details are available upon request.

Using expression (34) and rearranging terms yield:

$$\frac{knB'}{\lambda} = \frac{nV'\omega_g}{\lambda}\left(\frac{1}{1+\left(\frac{g_t}{l}-n\right)Tl'\omega_g+\left(\frac{g_t}{nl}-1\right)\theta\pi_g+\frac{(1+t_T)G_t}{knl}}\right), \tag{39}$$

where (11) and (16) have been used. Expression (39) relates the MCP of G at federal level (MCP_F^G) to the MCP of g at state level (MCP_S^g) when the former government behaves as Stackelberg leader and the latter one as follower. Note that if tax bases are not shared and the provision of public inputs corresponds to the central government exclusively, i. e., $t = g_t = G_t = 0$ and $\theta = 1$, expression (39) trivially becomes

$$\frac{knB'}{\lambda} = \frac{nV'\omega_g}{\lambda}\left(\frac{1}{1-nTl'\omega_g-\pi_g}\right), \tag{40}$$

that is, the relation between the MCP of G and the MCP of g at second best optimum in an unitary country.

Given these two alternative relationships between the MCP under different scenarios, a discussion can be initiated about if federal government is able to achieve the second best solution. Let $\eta = \frac{MCP_F^G}{MCP_S^g}$ be the variable that relates both MCP assuming Stackelberg approach. The relevant issue here is to know what extent this variable is with respect to 1; this way, we will know whether the federal structure of the country leads to an under or overprovision of the public input, using the unitary solution as benchmark.

Proposition 4 *If federal government plays as Stackelberg leader (with $T^* > 0$) and $\pi_g \gtrless 0$, then $\eta \lesseqgtr 1$. Hence, MCP_F^G may be higher, equal or smaller than MCP_S^g.*

Proof. Using (16) and rearranging terms, the expression in parenthesis in equation (39), i. e., the ratio η can be rewritten as follows:

$$\frac{1}{1+\frac{g_T}{knl}\left[G_g+(1+t_T)\,G_t\right]} \tag{41}$$

By (6) and (9), $g_T < 0$; if $\pi_g \gtrless 0$, then $G_g > 0$ when $T^* > 0$, and $G_t < 0$ by (9), $\forall T^* > 0$. As $1+ t_T \lesseqgtr 0$, we are not sure if the denominator of (41) is higher, equal or smaller than 1. So $\eta \lesseqgtr 1$. ■

Proposition 4 questions the ability of the federal government to achieve the second best optimum with no vertical grants. Notice that in an unitary country, also with $\tau > 0$ and $\pi_g \gtrless 0$, the MCP of G is higher than the MCP of g unambiguously (Proposition 1). From Proposition 4 a necessary condition to ensure the second best optimum must be established:

Corollary to Proposition 4 *Federal government that plays as Stackelberg may achieve the second best outcome if, and only if,* $1+ t_T > 1$, or what is the same, $t_T > 0$.

Proof. As the necessary condition for achieving an optimal result is that (41) to be bigger than 1, and since $g_T < 0$, $G_g > 0$, and $G_t < 0$, we need to have $G_g + (1 + t_T) G_t > 0$. Inserting here the expressions (5), (6), (8), (9), and the optimal federal tax rate T^* (33), it can be seen that $1+ t_T > 1$ is required to obtain that expression (41) to be bigger than one. Number of households has been normalised to 1 for making easier the proof. ■

As can be seen, the key point to internalize vertical externalities is the states' reaction function. We need to have state governments that increase their taxes when the federal government sets higher tax rates, and vice versa; only this way the federal policy-makers acting as Stackelberg can correct vertical externalities. One of the main implications is that the effectiveness of federal policy depends crucially on an empirical issue because the sign of t_T is theoretically ambiguous.

6 Concluding remarks

Sharing tax instruments between federal and subnational governments is a common feature in federations. It allows that different levels of government to be involved in financing their own public expenditures. However, the concurrency of tax power on the same tax base causes that vertical tax externalities appear, and a deviation of the results from the second best allocation is produced.

Also vertical externalities arise when public spending provided by one level of government affects other government's decisions. This is the case, for instance, of public inputs such as public investment, education and so on, that may exert different impacts on the tax revenues belonging to other governments. This second vertical externality has received very little attention in the literature on fiscal federalism, though it may be found in countries such as United States, Australia and Spain, or in supranational structures as can be seen with European regional policies.

This paper presents a model in which federal and state governments set per unit taxes on labor to finance two types of public expenditures. Federal government provides a consumption public good, while state governments supply a productivity-enhancing public input. Second best allocation is reached in an unitary country, and used as benchmark. When Nash behavior is to be assumed for governments, a vertical externality arises from the provision of public inputs, as well as the tax externality. While the former exerts an ambiguous effect on the federal tax revenues, the latter presents a clear negative influence. In this model, the sign and extent of the expenditure externality depend on the tax externality, amongst other things. Here, it has been proved that using production efficiency condition as criterion of optimality in federal systems leads to incorrect conclusions. Moreover, our results drive to distinguish between the cost of the public funds and the provision cost of the public input, which includes the former and the tax revenue effect.

Also the ability of federal government to achieve the second best outcome has been studied. Our approach restricts the policy instruments of the federal government, and it means that vertical transfers are not available for efficiency purposes. With the model described here, we cannot ensure that the federal government behaving as Stackelberg leader may attain the second best result. We only will have some guarantees about that when the states' reaction function to be such that an increase in the federal tax rate is followed by an increment in the state tax rate, and vice versa. Other result we find is that the optimum federal tax rate has not to be necessary negative in order to correct both vertical externalities.

Further research can be initiated on the basis of this paper. One interesting point would come from introducing households mobility in an environment with heterogeneous regions. It would affect efficiency of the equilibria, which would have to be restricted in order to avoid multiple solutions. Moreover, horizontal externalities would arise and the set of policy instruments probably should be enlarged to consider transfers between governments; otherwise, replicating the second best outcome may become impossible. A second extension could study behaviors of governments when the public goods and inputs they provide are substitutes or complementaries between themselves. Not only new vertical expenditures externalities would appear, but also new possibilities for correcting externalities. Indeed, federal government could use its own public spending with the aim of providing an optimal amount of public inputs. Performance of this policy would be on the basis that different layers of government are able to provide the same o similar public inputs. Thirdly, given the relevance of states' reaction function on the effectiveness of federal policies, empirical researches could highlight how the state governments modify their behaviors when facing federal decisions. To the best of our knowledge, there is a stimulating lack of empirical papers on this issue. Papers such as Besley and Rosen (1998), Esteller-More and Sole-Olle (2001) or Anderson et al. (2004) could be enlarged to deal explicitly with issues related to the interplays between expenditure and tax externalities or the MCP. At this point, the empirical analyses should consider not only the $MCPF$ (in fact, a not very usual aspect regarded in this kind of approaches), but also the tax revenue effect arising when there exist complementarities between public spending and tax revenues.

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